



Wednesday, September 19th

Dmitry Kramkov - "A Model for a Large Investor Trading At Market Indifference Prices":

We present an equilibrium-based model for large economic agent where she trades with the market makers at their utility indifference prices. We perform an asymptotic analysis of this model for the case of "small" investor and derive as a result a liquidity correction to the prices of non-traded derivatives. The presentation is based on a joint project with Peter Bank.

Mete Soner - "Second Order BSDE's: New Results on Existence":

The theory of BSDE's have been well developed and had many applications in several fields as well as in finance. In joint work with Cheredito, Touzi and Victoir, we have extended the theory to include "second order" terms. In this talk, I will outline the general theory. Then, I will describe a new weak approach to the question of existence and some results on the numerical solutions.

Esko Valkeila - "Approximation of Geometric Fractional Brownian Motion":

We give an approximation to geometric fractional Brownian motion in the sense of weak convergence, in the case when the self-similarity index of driving fractional Brownian motion is bigger than one half. Our approximation has the advantage to the previous ones in that the associated pricing model is free of arbitrage opportunities and complete.

Tomas Björk - "Optimal Investments Under Partial Information":

In this paper we consider optimal investment problems where the local rate of returns of the assets under consideration cannot be observed directly. The problem is that of finding the optimal portfolio strategy which maximizes the utility of terminal wealth. The constraint is that the portfolio has to be adapted to the filtration generated by observations of the asset price processes. This leads to an optimal control problem under partial information. For various special cases, problems of this kind has earlier been successfully studied by Brennan, Baeuerle, Rieder, Brendle, Nagai, Runggaldier and others. The contribution of the present paper is that we study a fairly general problem without any assumptions of a Markovian structure. Within this framework we obtain surprisingly explicit formulas for optimal wealth and optimal portfolio strategies. The existing results in the literature then come out as special cases of the general theory. (Joint work with Mark Davis and Camilla Landen)

Thaleia Zariphopoulou - "Investment Performance Measurement, Risk Tolerance and Optimal Portfolio Choice":

A new approach to measure the dynamic performance of investment strategies is introduced. To this aim, a family of stochastic processes defined on $[0, \infty)$ and indexed by a wealth argument is used. Optimality is associated with their martingale property along the optimal wealth trajectory. The optimal portfolios are constructed via stochastic feedback controls that are functionally related to differential constraints of fast diffusion type. A multi-asset Ito type incomplete model is used.



Session 1

Jan Obloj - "Completing Market Using Options: Necessary and Sufficient Conditions":

We consider the question of market completeness when we can use for hedging the underlying together with some (liquidly traded) options. We follow the general approach of Romano and Touzi (1997) and Davis (2004). We assume a general multidimensional diffusion model (in particular a stochastic volatility model). We do not specify the dynamics of options prices exogenously but assume we work under the pricing measure so that assets' prices are given as discounted conditional expectations of their payoffs. We are interested in the following question: can we achieve any contingent claim as a final value of a trading strategy in the underlying and the given set of liquidly traded options? We give a necessary and sufficient condition for this to hold which generalizes upon sufficient conditions given in the previous works. We then extend this to jump-diffusion models. (Joint work with Mark Davis.)

Mihail Zervos - "A Model for Reversible Investment Capacity Expansion":

We consider the problem of determining the optimal investment level that a firm should maintain in the presence of random price and/or demand fluctuations. We model market uncertainty by means of a geometric Brownian motion, and we consider general running payoff functions. Our model allows for capacity expansion as well as for capacity reduction, with each of these actions being associated with proportional costs. The resulting optimisation problem takes the form of a singular stochastic control problem that we solve explicitly. We illustrate our results by means of the so-called Cobb-Douglas production function. The problem that we study presents a model, the associated Hamilton-Jacobi-Bellman equation of which admits a classical solution that conforms with the underlying economic intuition but does not necessarily identify with the corresponding value function, which may be identically equal to infinity. Thus, our model provides a situation that highlights the need for rigorous mathematical analysis when addressing stochastic optimisation applications in finance and economics, as well as in other fields.

Giovanni Barone-Adesi - "Barrier Option Pricing Using Adjusted Transition Probabilities":

In the existing literature on barrier options, much effort has been exerted to ensure convergence through placing the barrier in close proximity to, or directly onto, the nodes of the tree lattice. In this paper we show that this may not be necessary to achieve accurate option price approximations. Using the Cox/Ross/Rubinstein binomial tree model and a suitable transition probability adjustment we demonstrate that our "probability-adjusted" model exhibits increased convergence to the analytical option price. We study the convergence properties of various types of options including (but not limited to) double knock-out, exponential barrier, double (constant) linear barriers and linear time-varying barriers. For options whose strike price is close to the barrier we are able to obtain numerical results where other models fail and, although convergence tends to be slow, we are able to calculate reasonable approximations to the analytical option price without having to reposition the lattice nodes.

Alexandra Dias - "Semi-parametric Estimation of Portfolio Tail Probabilities":

In this paper we estimate the probability of occurrence of a large portfolio loss. This accounts for estimating the probability of an event in the far joint-tail of the portfolio loss distribution. These are rare extreme events and we use a semi-parametric procedure from extreme value theory to estimate its probability. We find that the univariate loss distribution is heavy tailed for three market indexes and that there is dependence between large losses in the different indexes. We estimate the probability of having a large loss in a portfolio composed by these indexes and compute the portfolio components weights which minimize the probability of a large portfolio loss. With this procedure we are able to estimate the probability of portfolio losses never incurred before without the need of specifying a parametric dependence model and where increasing the number of portfolio components does not bring complications to the estimation.

Erika Hausenblas - "Existence, Uniqueness and Regularity of Parabolic SPDEs Driven by Poisson Random Measure":

The topic of the talk are stochastic evolutions equations driven by a Poisson random measure and will be based on my works 1, 2 and 3. First, the Poisson random measure will be introduced. Then stochastic evolutions equations, short SPDEs, driven by a Poisson random measure will be discussed. Here, existence and uniqueness results will be presented and integrability properties and the càdlàg property will be outlined. We will close with some typical example of such SPDEs.



Session 2

Florian Kramer - "Risk and Valuation of Mortality Contingent Catastrophe Bonds":

Catastrophe Mortality Bonds are a recent capital market innovation providing insurers and reinsurers with the possibility to transfer catastrophe mortality risk off their balance sheets to capital markets.

We introduce a time-continuous model for analyzing and pricing catastrophe mortality contingent claims based on stochastic modeling of the force of the mortality. The model consists of two components: a baseline component governed by a diffusion reflecting the "regular" fluctuations of mortality over time and a catastrophe component modeled by a Non-Gaussian Ornstein-Uhlenbeck process representing catastrophic events.

Historical and risk-adjusted parameterizations of the proposed model based on three different calibration procedures are derived. The resulting loss profiles and prices are compared to loss profiles provided by the issuers and to market prices, respectively. We find that the profiles are subject to great uncertainties and should hence be considered with care by investors and rating agencies. (Joint work with Daniel Bauer)

Stéphane Crépey - "About the Pricing Equation in Finance":

We establish the well-posedness of reflected BSDE problems with jumps grown out from pricing problems in finance, and we derive the related variational inequality approach. We first construct a rather generic Markovian model made of a jump diffusion X interacting with a pure jump process N (which in the simplest case reduces to a Markov chain in continuous time). The jump process N defines the so-called regime of the coefficients of X , whence the name of Jump-Diffusion Setting with Regimes for this model. Motivated by optimal stopping and optimal stopping game problems (pricing equations of American or Game Contingent Claims, in terms of financial applications), we introduce the related reflected and doubly reflected Markovian BSDEs, showing that they are well-posed in the sense that they have unique solutions, which depend continuously on their input data. We then introduce the system of partial integro-differential obstacles problems formally associated to our reflected BSDE problems. We show that the state-processes (first components Y) of the solutions to our reflected BSDEs can be characterized in terms of the value functions to the related optimal stopping or game problems, given as viscosity solutions with polynomial growth to the related obstacles problems. We further establish a discontinuous viscosity semisolutions comparison principle (or maximum principle) for our problems, which implies in particular uniqueness of viscosity solutions for these problems. This maximum principle is subsequently used for proving the convergence of stable, monotone and consistent approximation schemes to our value functions.

Ghulam Sorwar - "Valuation of Two-Factor Interest Rate Contingent Claims Using Green's Theorem":

Over the years a number of two-factor interest rate models have been proposed that have formed the basis for the valuation of interest rate contingent claims. This valuation equation often takes the form of a partial differential equation, that is solved using the finite difference approach. In the case of two factor models this has resulted in solving two second order partial derivatives leading to boundary errors, as well as numerous first order derivatives. In this paper we demonstrate that using Green's theorem second order derivatives can be reduced to first order derivatives, that can be easily discretised; consequently two factor partial differential equations are easier to discretise than one factor partial differential equations. We illustrate our approach by applying it to value contingent claims based on the two factor CIR model. We provide numerical examples which illustrate that our approach shows excellent agreement with analytical prices and the popular Crank Nicolson method.

Birgit Rudloff - "Hedging in Incomplete Markets with Convex Risk Measures":

In incomplete financial markets not every contingent claim can be replicated by a self-financing strategy. Starting with an amount of money smaller than the superhedging price of the claim, we want to find a strategy that minimizes the risk of the shortfall. We use a convex risk measure. This problem can be split into a static optimization problem and a representation problem. We show that the optimal strategy consists in superhedging a modified claim whose payoff is the product of the solution of the static problem and the original payoff. To solve the static problem we apply convex duality methods. We provide necessary and sufficient optimality conditions.

Andreas Hamel - "Set-Valued Risk Measures":

Jouini et al. (Finance & Stochastics 8, 2004) proposed the concept of set-valued coherent risk measures in order to incorporate market frictions like transaction costs, liquidity bounds etc. In the evaluation of the risk of a portfolio consisting of several assets. We extend this concept to general risk measures with values in the set of all subsets of a finite dimensional space being set-valued translative and monotone functions. For the general case, we establish primal representation results in terms of acceptance sets. For the convex case, we give a complete duality theory



parallel to the scalar case including a penalty function representation. This theory is based on extensions of Convex Analysis to set-valued functions which are also new – including definitions of Fenchel conjugates for set-valued functions and a corresponding biconjugation theorem. Another natural question is: How shall we select a single risk canceling element out of a specific value (a set!) of a risk measure in order to cancel the risk of the portfolio under consideration? the answer will be found via a scalarization concept that has an interpretation in terms of prices in one of several possible currencies. A list of examples is given along with "standardized" procedures (primal and dual) how a known scalar risk measure can be transformed into a set-valued one. In many cases, there is more than one generalization (more or less risk averse) of one scalar risk measure. We shall present set-valued counterparts for (negative) expectation, VaR and AVaR, the (negative) essential infimum and the entropy measure.



Session 3

Stefanie Kammer - "Credit Spread Volatility Under a First Passage Time Model":

A credit default swap (CDS) offers protection against default of a reference entity. Therefore the protection buyer regularly pays an insurance fee, the credit spread, but only as long as no default has happened. In case of default before contract maturity, the protection seller pays a claim amount as agreed to the protection buyer. By now CDSs are the most liquid credit-risky market instruments. Considering CDS contracts with various times to maturity leads to a whole credit spread term structure. Credit spread curves vary stochastically over time in level and shape. A model that reflects credit spread dynamics is crucial for pricing any derivatives on credit spread (such as credit baskets, credit spread options and CDOs), especially when considering longer maturities. Up to now credit spread models are adapted to the market's credit spread term structure only, but do not integrate credit spread volatility. For example, the deterministic time change model by Overbeck & Schmidt [1] can perfectly fit the credit spread curve, but contradicts empirical behavior of credit spread volatility. Our concern is not a perfect fit of today's market curve, but a model that can be adapted to both, credit spread term structure and credit spread volatility. Within a structural approach we consider the general class of stochastic time change models where Brownian motion is time changed by an absolutely continuous process. We derive an analytical first passage time distribution (FPTD) for the one-dimensional model and also for the two-dimensional model under additional asset correlation, by applying a result of Zhou [2]. For the multi-dimensional model a FPTD is yielded under a simpler dependence structure arising from the same time change. Using our analytical FPTD we can derive credit spread dynamics via Itô's rule. Our joint and multivariate default probabilities can be applied for pricing credit derivatives that dependent on several names. FPTDs are also important in other applications such as barrier option pricing. We provide an example of a specific time change model and show how it can be calibrated to credit spread term structure and credit spread volatility.

Christian Schmidt - "Outperforming Benchmarks in Fixed-Income Markets":

We consider a fixed-income market where the market dynamics are driven by a state-process and where the market-price of risk is allowed to be stochastic. In this market, a benchmark process is defined by means of a trading strategy in fixed income securities. The investor is interested in maximising expected utility of terminal wealth relative to this benchmark.

We propose to decompose the trading strategy into two parts: the benchmark strategy which provides the baseline and an active strategy which considers deviations relative to the benchmark and which will lead to the overall optimal strategy for the optimization problem. This decomposition allows for a nice interpretation and even simplifies computation in some cases.

The optimal portfolio strategy is derived from a Hamilton-Jacobi-Bellman equation. For an affine market-model, we provide analytical results (up to ODEs) and analyse the influence of the benchmark on the optimal strategy. We conclude with a discussion of possible extensions towards regime-switching models.

Linus Kaisajuntti - "An N-Dimensional Markov-Functional Model":

This paper develops an n-dimensional LIBOR Markov-functional interest rate model using in effect the same techniques as for lower-dimensional Markov-functional models, but under a slightly different setup. This means formulating the model using forward induction under the Spot measure instead of backward induction under the Terminal measure and using the Monte Carlo method instead of more efficient numerical integration and lattice methods. However, despite the use of the Monte Carlo method it turns out that the proposed n-dimensional Markov-functional model is significantly more efficient than its LIBOR market model counterpart and is very well suited for certain type of path-dependent derivatives. Moreover, the n-dimensional Markov-functional model provides a powerful framework for analysing Markov-functional models. In addition to Bennet & Kennedy (2005), who shows that one-factor LIBOR market models and one-dimensional Markov-functional models are very similar, we perform tests comparing the n-dimensional versions over a variety of market conditions. The tests confirm major similarities and we argue that the intuition gained from the LIBOR market model SDE will always be applicable, irrespective of dimensions.

Jan Maruhn - "Robustifying Static Hedges for Barrier Options Against Dynamics of the Volatility Surface":

Since static hedge portfolios for barrier options consist of several standard options, these portfolios are strongly exposed to movements of the volatility surface over time. In this talk we present a new static super-replication approach which leads to robust portfolios guaranteeing the hedge performance for an infinite number of future volatility surface scenarios. As it turns out, the hedge can be computed by solving a suitable semi-infinite



optimization problem. After proving existence-, convergence- and duality results, we apply the method to real world data and obtain market-typical super-hedges with surprisingly low cost.

Natalie Packham - "Modelling Credit-Spread Dynamics in a Hitting-Time Model":

Standard hazard rate models for credit default times capture jumps to default, but in general, we expect a credit to deteriorate over time before it defaults. This should be reflected in a credit model, as, for instance, the valuation of some credit instruments depends on jumps of the underlying credit spread (an example is the Leveraged Credit-Linked Note). Furthermore, the valuation of options on credit derivatives requires the specification of the underlying credit derivative's dynamics through time. We present a tractable hitting-time model that is based on a Wiener process with a stochastic time change. The time change is a Lévy-driven Ornstein-Uhlenbeck process, which reflects the observation that credit spreads move up sharply and then tend back gradually over time. We present an efficient technique for computing default probabilities and conditional default probabilities numerically. The calibration to market-given data is investigated with a particular focus on the economic interpretation of the parameters involved (Joint work with Lutz Schlögl, Quantitative Credit Research, Lehman Brothers).



Session 4

Jean-François Chassagneux - "Discrete-Time Approximation of American Option and Game Option Price": American Option prices and Game Option prices can be represented respectively by simply reflected BSDEs (El Karoui et al. 1997) and doubly reflected BSDEs (Cvitanic and Karatzas, 1996). We then study the discrete time approximation of the solution (Y, Z, K) of such equations. As in Ma and Zhang (2005), we consider a Markovian setting with reflecting barriers of the form $h(X)$ where X solves a forward SDE. We first focus on the discretely reflected case. Based on a new representation for the Z component, which is directly linked to the Delta of the options, we retrieve the convergence result of Ma and Zhang (2005) without their uniform ellipticity condition on X . These results are then extended to the case where the reflection operates continuously both for American Options and Game Options. We also improve the bound on the convergence rate when $h \in C^2_b$ with Lipschitz second derivative.

Christina Erlwein - "Filtering and Optimal Parameter Estimation of a Hidden Markov Model for Electricity Spot Prices":

We develop a hidden Markov model for electricity spot price dynamics, where the spot price follows an exponential Ornstein-Uhlenbeck process with an added compound Poisson process. This way, the model allows for mean-reversion and possible jumps. All parameters are modulated by a hidden Markov chain in discrete time. They are therefore able to switch between different economic regimes representing the interaction of various hidden factors. Through the application of reference probability technique, adaptive filters are derived, which in turn, provide optimal estimates for the state of the Markov chain and related quantities of the observation process. The EM algorithm is applied to find optimal estimates of the model parameters in terms of the recursive filters. We implement this self-calibrating model on a deseasonalized series of daily spot electricity prices from the Nordic exchange Nord Pool. On the basis of one-step ahead forecasts, we found that the model is able to capture the stylised features of Nord Pool spot prices. This is joint work with Fred Espen Benth (CMA, University of Oslo, Norway) and Rogemar Mamon (University of Western Ontario, London, Canada)

Torsten Schöneborn - "Dynamic Optimal Execution Strategies and Predatory Trading":

We consider a risk-averse investor holding a large asset position in an illiquid market. When selling this position, the investor faces a trade-off between high costs of quick liquidation and high uncertainty of slow liquidation. Most previous research on optimal liquidation strategies focused on static strategies and mean-variance risk-averse investors. We generalize these investigations to include dynamic liquidation strategies and investors seeking von Neumann-Morgenstern utility maximization. In this setting, we find that investors with constant absolute risk aversion will pursue static strategies, while investors with increasing or decreasing absolute risk aversion will adjust their trading speed to exogenous changes in market prices. Furthermore, we extend the analysis to the case where competing market participants are aware of the investor's trading intentions. We show that the liquidity characteristics and the number of competitors in the market determine the optimal strategy for the competitors: they either provide liquidity to the seller, or they prey on him by simultaneous selling. If they provide liquidity, it might be sensible for the seller to pre-announce a trading intention ("sunshine trading"). (Joint work with Alexander Schied)

Koichi Matsumoto - "Mean-Variance Hedging in an Illiquid Market":

We study a hedging problem of a contingent claim in a discrete time model, where the contingent claim is hedged by one illiquid risky asset. In our model, the investor cannot always trade the illiquid asset, though he can always observe or estimate the price of the illiquid asset. In other words, the trade times are not only discrete but also random. Our model is a kind of random trade time model studied by Rogers and Zane (2002) and Matsumoto (2003, 2006). In this setting the perfect hedging is difficult and then we measure the hedging error by a quadratic criterion. An outline of our study is as follows. First we fix the initial conditions. We show the existence of the optimal hedging strategy, and we give the optimal strategy as a recursive formula. Secondly we consider the optimization of the initial conditions. In the illiquid market, it is not easy to change the portfolio and then the initial conditions are important. We express the optimal initial conditions simply, using the signed measure. Finally we consider a one-period binomial model by way of example.

Beatrice Acciaio - "Optimal Risk Allocation when Agents have Different Reference Probability Measures":

We study the problem of optimal sharing of an aggregate risk between agents whose preferences are represented by cash-invariant convex functionals. Under the hypothesis of law-invariance with respect to a given probability



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measure, the existence of optimal solutions is proved by Jouini, Schachermayer and Touzi (2005) and Acciaio (2006) (on the space of essentially bounded financial positions) and by Filipovic and Svindland (2007) (on L_p , for any p in $[1, \infty]$). Here we consider choice functionals which are law-invariant with respect to different probability measures. In this context we show the exactness of the optimal allocation problem on L_p , for any p in $(1, \infty)$. Moreover, we provide a representation result for the convolution of such functionals on the space of essentially bounded financial positions.



Session 5

Jan Palczewski - "On the Wealth Dynamics of Self-financing Portfolios under Endogenous Prices":

In this paper we study market selection and survival of self-financing trading strategies in a continuous-time market model with endogenous prices. This model builds on the common continuous-time model in mathematical finance. The prices however are derived from the demand and supply of traders, whose decisions are influenced by an exogenous dividend process. Our approach promotes an evolutionary point of view that abstracts from utility functions. It borrows from a common knowledge of economic theory that states that market pressures eventually select those traders who are better adapted to the prevailing conditions. The main result is on the survival of trading strategies. We show that there is a single surviving trading strategy among all constant strategies. Traders following this strategy eventually accumulate all the wealth of the market. Our findings are surprising in the view of earlier studies of related models with discrete time as our model lacks exponential stability and cannot be treated by well-developed theory of Stochastic Dynamical Systems. This paper is a joint work with Jesper Lund Pedersen (Copenhagen) and Klaus Reiner Schenk-Hoppé (Leeds).

Amal Merhi - "Irreversible Capacity Expansion with Proportional and Fixed Costs":

We consider the problem of determining the optimal capacity expansion strategy that a firm operating within a random economic environment should adopt. We model market uncertainty by means of a geometric Brownian motion. The objective is to maximise a performance criterion that involves a general running payoff function and associates a fixed and a proportional cost with each capacity increase. The resulting optimisation problem takes the form of a two-dimensional impulse control problem that we explicitly solve.

Ralf Werner - "Consistency of Robustified Portfolio Optimization Frameworks":

In recent years, several alternatives to the traditional Markowitz portfolio optimization framework gained more and more popularity. Among those, the most prominent ones are probably Michaud's resampling approach and the robust counterpart ansatz. For clarity, the presentation will focus on the general mathematical framework for Michaud's setup, which includes traditional Markowitz portfolio optimization as a special case. As main result we will show that this approach is consistent in a statistical sense, i.e. The estimated portfolios converge to the true optimal portfolios if consistent point estimators are used for input data estimation. As the proof relies on continuity properties of the solution of parametric convex conic optimization problems, it allows for very general portfolio constraints. These novel findings do not only provide a justification for Michaud's approach but also clearly extend existing results for the traditional Markowitz model. Finally, we will sketch analogous results for the robust counterpart ansatz. (Joint work with Katrin Schoettle, TU Muenchen)

Jostein Paulsen - "Optimal Dividend Payments and Reinvestments of Diffusion Processes with Both Fixed and Proportional Costs":

Assets are assumed to follow a diffusion process subject to some conditions. The owners can pay dividends at their discretion, but whenever assets reach zero, they have to reinvest money so that assets never go negative. With each dividend payment there is a fixed and a proportional cost, and so with reinvestments. The goal is to maximize expected value of discounted net cash flow, i.e. dividends paid minus reinvestments. It is shown that there can be two different solutions depending on the model parameters and the costs.

1. Whenever assets reach a barrier they are reduced by a fixed amount through a dividend payment, and whenever they reach 0 they are increased by a fixed amount by a reinvestment.
2. There is no optimal policy, but the value function is approximated by policies of the form described in Item 1 for increasing barriers. We provide criteria to decide whether an optimal solution exists, and when not, show how to calculate the value function. It is discussed how the problem can be solved numerically and numerical examples are given.