MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

T a g u n g s b e r i c h t 51/1992

Large Deviations and Applications

29. 11. bis 5. 12. 1992

Zu dieser Tagung unter der gemeinsamen Leitung von E. Bolthausen (Zürich), J. Gärtner (Berlin) und S. R. S. Varadhan (New York) trafen sich Mathematiker und mathematische Physiker aus den verschiedensten Ländern mit einem breiten Spektrum von Interessen.

Die Theorie vom asymptotischen Verhalten der Wahrscheinlichkeiten großer Abweichungen ist einer der Schwerpunkte der jüngeren wahrscheinlichkeitstheoretischen Forschung. Es handelt sich um eine Präzisierung von Gesetzen großer Zahlen. Gegenstand der Untersuchungen sind sowohl die Skala als auch die Rate des exponentiellen Abfalls der kleinen Wahrscheinlichkeiten, mit denen ein untypisches Verhalten eines stochastischen Prozesses auftritt. Die Untersuchung dieser kleinen Wahrscheinlichkeiten ist für viele Fragestellungen interessant.

Wichtige Themen der Tagung waren:

- Anwendungen großer Abweichungen in der Statistik
- Verschiedene Zugänge zur Theorie großer Abweichungen
- Stochastische Prozesse in zufälligen Medien
- Wechselwirkende Teilchensysteme und ihre Dynamik
- Statistische Mechanik, Thermodynamik
- Hydrodynamischer Grenzübergang
- Verhalten von Grenzflächen, monomolekulare Schichten
- Dynamische Systeme und zufällige Störungen
- Langreichweitige Wechselwirkung, Polymere
- Stochastische Netzwerke

Die Tagung hatte 47 Teilnehmer, es wurden 42 Vorträge gehalten.

Abstracts

G. Ben Arous (joint work with A. Guionnet)

Langevin dynamics for spin glasses

We study the dynamics for the Sherrington-Kirkpatrick model of spin glasses. More precisely, we take a soft spin model suggested by Sompolinski and Zippelius. Thus we consider diffusions interacting with a Gaussian random potential of S-K-type.

(1) We give an "annealed" large deviation principle for the empirical measure at the process level. We deduce from this an annealed law of large numbers and a central limit theorem. The limit process is a new object, a nonlinear and non-Markovian process.
(2) We then show how for certain initial measures this can give the "quenched" law of large numbers and we see again that the disorder in the interaction produces non-Markovianity of the limit.

All this is valid only above a critical temperature on a given interval of time or before a critical time at a given temperature.

Anton Bovier (joint work with V. Gayrard and P. Picco)

Thermodynamics of the Hopfield model

We study the Hopfield model of a neural network in the spirit of disordered mean field models of spin systems. The disorder here resides in the coupling matrix $J_{ij} = (1/N)\sum_{\mu=1}^{m} x_i^\mu x_j^\mu$, where $\{x_i^\mu\}$ is a family of i. i. d. random variables taking the values $+1$ and -1 with equal probability. Properties of this model depend crucially on the parameter m. We present the following results:

(1) In the cases where $m/N \searrow 0$, as $N \nearrow \infty$, we prove that the free energy of this model converges to that of the standard Curie-Weiss model, almost surely. Moreover, we show that to each of the vectors x^μ there corresponds, for $T < 1$, Gibbs measures in the infinite volume limit that are concentrated on configurations having overlap $a(T)$ with the vector x^μ, and overlap zero with all other vectors x^ν, $\nu \neq \mu$.
(2) In the case where $m/N = \alpha$, with α sufficiently small, we show that the structure of the Gibbs measures remains the same as before, for $T \leq T(\alpha) < 1$, where $T(\alpha) \to 1$ as $\alpha \searrow 0$.

Francis Comets

Erdös-Rényi laws and Gibbs measures

Erdös-Rényi type of laws state that in a given sample of size n, one will observe in subsamples of size $(1/t)\log n$ all deviations with rate of decay less or equal to t $(t > 0)$, with probability 1 as $n \to \infty$.

(1) We give general formulations of this result, for the empirical field or process under the condition of uniform large deviation estimates (or hypermixing processes).

(2) We give applications to Gibbs measures, and we study in this case the limit $t \searrow 0$. The result then yields positive answers to questions like: Can we detect phase transition from a single (but large) sample? Can we learn some information on the other Gibbs measures?

Ted Cox (joint work with Andreas Greven and Tokuzo Shiga)

Finite and infinite systems of interacting diffusions

The subject of this talk is a theorem relating the asymptotic behavior of large finite systems of interacting diffusions and the corresponding infinite system. The infinite system $x(t) = \{\, x_i(t),\, i \in \mathbb{Z}^d \,\}$ is the Markov process determined by

$$dx_i(t) = \left[\sum_{j\in\mathbb{Z}^d} a(i,j)x_j(t) - x_i(t)\right]dt + \sqrt{g(x_i(t))}\, dW_i(t) \qquad (*)$$

where $a(i,j)$ is an irreducible random walk kernel on $\mathbb{Z}^d$, $g\colon [0,1] \to \mathbb{R}^+$ is Lipschitz, $g(0) = g(1) = 0$, $g > 0$ on $(0,1)$, and $\{W_i(t)\}$ is a family of independent Brownian motions. There is a family $\{\, \nu_\theta,\, \theta \in [0,1] \,\}$ of invariant measures for $x(t)$ with $E^{\nu_\theta} x_i = \theta$. The finite systems $x^N(t) = \{\, x_i^N(t),\, i \in (-N,N]^d \,\}$ are defined by an equation like $(*)$ treating $(-N,N]^d$ as a torus. The main result is that under some conditions, for $t_N \uparrow \infty$ with N, $t_N/(2N)^d \to s \in [0,1]$,

$$\mathcal{L}(x^N(t_N)) \Rightarrow \int_{[0,1]} Q(\varrho, d\theta)\, \nu_\theta$$

where $Q(\varrho, \cdot\,)$ is the transition of a certain diffusion on $[0,1]$. In particular, we see that if $t_N = o(N^d)$ as $N \to \infty$ then $\mathcal{L}(x^N(t_N)) \Rightarrow \nu_\varrho$, so that the invariant measures of the infinite system describe the behavior of the finite systems for times up to a certain order.

P. Dai Pra

Large deviations for interacting particle systems

We study the large deviation of the space-time empirical averages of a d-dimensional stochastic spin system whose Markov semigroup is generated by the operator

$$Lf(\sigma) = \sum_{i \in \mathbb{Z}^d} c(\vartheta_i \sigma)[f(\sigma^i) - f(\sigma)]$$

where ϑ_i is the shift on $\{-1,1\}^{\mathbb{Z}^d}$ and $\sigma^i(j) = (-1)^{\delta_{ij}} \sigma(j)$. We prove a n^{d+1}-large deviation principle for the empirical process

$$R_{n,\omega} = \frac{1}{n^{d+1}} \sum_{i \in \{0,1,\dots,n-1\}^d} \int_0^n \delta_{\vartheta_{t,i}\omega} \, dt,$$

where $\omega \in \Omega = \mathbb{D}(\mathbb{R}, \{-1,1\}^{\mathbb{Z}^d})$ and $\vartheta_{t,i}$ are the space-time shift maps on Ω, and we identify the rate function. Moreover, we prove that the zeros of the rate function correspond to the invariant measures for the system. We also give results on some related problems, as the "contraction" to deviations of lower level and critical large deviations for non-ergodic systems.

Donald A. Dawson

Some comments on the hierarchical mean-field limit

We begin with a system of a large number of components where interactions are organized in a hierarchical manner. The kth level of the hierarchy is comprised of N objects of the $(k-1)$st level and the strength of the interaction decreases as a function of the hierarchical distance (and also as a function of N). The single level hierarchy in the limit $N \to \infty$ is known as the mean-field limit. The case in which N is fixed and $k \to \infty$ corresponds to the thermodynamic limit. The hierarchical mean-field limit corresponds to the finite or infinite hierarchy in the $N \to \infty$ limit. The effect of taking the limit $N \to \infty$ is to separate the natural time scales or spatial scales relevant to the different levels of the hierarchy. To illustrate this we consider two examples. The first is the continuous spin ferromagnetic model. In joint work with Jürgen Gärtner this hierarchical mean-field limit of this ferromagnetic model is analysed using multilevel large deviation theory as $N \to \infty$. This analysis leads to a notion of discrete symmetry breaking in the mean-field limit. The second model considered is the stepping stone model arising in population genetics. This model has been analysed in joint work with Andreas Greven using multiple time scale analysis. This work shows that the criteria for continuous symmetry breaking in this model in the hierarchical mean-field and in the thermodynamic limit sense are in fact equivalent for a large family of interaction strengths.

Frank den Hollander (joint work with A. Greven)

Large deviations for a random walk in random environment

Let $\omega = (p_x)_{x\in\mathbb{Z}}$ be an i. i. d. collection of $(0,1)$-valued random variables. Given ω, let $(X_n)_{n\geq 0}$ be the Markov chain on $\mathbb{Z}$ defined by $X_0 = 0$ and $X_{n+1} = X_n \pm 1$ with probability p_{X_n} resp. $1 - p_{X_n}$. It is shown that X_n/n satisfies a large deviation principle, i. e.,

$$\lim_{n\to\infty} \frac{1}{n} \log P_\omega(X_n = \lfloor \theta_n n \rfloor) = -I(\theta) \quad \omega\text{-a. s. for any } \theta_n \to \theta \in [-1,1].$$

First we derive a representation of the rate function I in terms of a variational problem. Second we solve the latter explicitly in terms of random continued fractions. This leads to a classification and qualitative description of the shape of I. In the recurrent case I is non-analytic at $\theta = 0$. In the transient case I is non-analytic at $\theta = -\theta_c, 0, \theta_c$ for some $\theta_c \geq 0$, with linear pieces in between.

J.-D. Deuschel (joint work with A. Pisztora and C. Newman)

Critical large deviations

Let P_0 be a product measure on $\Omega = E^{\mathbb{Z}^d}$ and denote by $R_N(\omega) = (1/|V_N|)\sum_{k\in V_N} \delta_{\theta_k \omega}$ the empirical field of the box $V_N = [1,N]^d$. For a given interaction potential ϱ, define the approximate microcanonical distribution $\mu_{N,\delta}(\,\cdot\,) = P_0(\,\cdot \mid |U_N - \nu| \leq \delta)$, where U_N is the average energy of V_N. Large deviations show that the law of the empirical field R_N converges at a volume exponential rate on the set of Gibbs distributions at an appropriate inverse temperature $\beta = \beta(u)$. In case of phase transition, we expect that R_N concentrates on the extremal Gibbs states. We show that a surface exponential rate occurs for the Ising model. The central estimate is a surface-order large deviations for the empirical magnetization of the free boundary Gibbs distribution. The method uses F-K-percolation at sufficiently small temperature and the isoperimetric estimate.

Hermann Dinges

Second order large deviations

We start with a family of distributions $\{\mathcal{L}(X_\varepsilon)\colon \varepsilon \to 0\} = \{p_\varepsilon(x)\,dx\colon \varepsilon \to 0\}$ on $U \subset \mathbb{R}^d$ of the form (uniformly on compacts)

$$p_\varepsilon(x)\,dx = (2\pi\varepsilon)^{-d/2} \times \exp\Big(-\frac{1}{\varepsilon}K(x) - K_0(x) - \varepsilon K_1(x) + o(\varepsilon)\Big)dx_1\dots dx_d.$$

$K(\,\cdot\,)$ is not necessarily the Legendre-transform of a cumulant generating function. Just $K(\,\cdot\,)$ smooth and $K(x^*) = 0$ for some x^*, $K(x) > 0$ for $x \neq x^*$, $K''(\,\cdot\,)$ positive definite ($K_0(\,\cdot\,)$ and $K_1(\,\cdot\,)$ are required to satisfy certain smoothness conditions as well.) For nice sets $A = \{x\colon F(x) \le \text{const}\}$ we find an asymptotic expansion

$$\Lambda(\Pr(X_\varepsilon \in A)) = \frac{1}{\varepsilon}K(\hat{x}) + (H_0(\hat{x})) + \delta(\hat{x})) + O(\varepsilon),$$

where $\Lambda(p) = [\Phi^{-1}(p)]^2/2$, $K(\hat{x}) = \inf\{K(x)\colon x \in \partial A\}$, $\delta(\hat{x})$ vanishes when A is a halfspace, and

$$H_0(\hat{x}) = \frac{1}{2}\ln\Big[\frac{K'(K'')^{-1}K'}{2K}(\hat{x})\Big] + K_0(\hat{x}) + \frac{1}{2}\ln|\det K''(\hat{x})|.$$

In the second part of the lecture such an asymptotic expansion was given explicitly in a particular case; we studied an approximation of the so called non-central t-distribution

$$T^{(n)} := \frac{\overline{Y}}{\sqrt{\frac{1}{n-1}\sum_{i=1}^n (Y_i\overline{Y})^2}}$$

in the general case and in the special Gaussian case

$$\widetilde{T}^{(n)} = \frac{\vartheta + (1/\sqrt{n})Z_0}{\sqrt{(1/n)\sum_{i=1}^n Z_i^2}}$$

with $Z_0, Z_1, \dots, Z_n$ independent standard normal. Then

$$\Pr_\vartheta\Big(\widetilde{T}^{(n)} \le \frac{\vartheta + a}{\sqrt{1 - a(a+\vartheta)}}\Big) \approx \Phi\Bigg(\pm\sqrt{2}\sqrt{nK(\vartheta,a) - \frac{1}{2}\ln\Big[\frac{2K(\vartheta,a)}{a^2(1 + (1/2)\vartheta(a+\vartheta))}\Big] + \text{rest}}\Bigg)$$

where $2K(\vartheta,a) = a^2 - a(a+\vartheta) - \ln[1 - a(a+\vartheta)]$ for $a \in (-\infty, +\infty)$.

Richard S. Ellis (joint work with Paul Dupuis)

A stochastic optimal control approach to the theory of large deviations

We present a new and widely applicable approach to the theory of large deviations which is based on stochastic optimal control theory. In our opinion, this approach reduces many aspects of the theory of large deviations to the theory of weak convergence of probability measures. We demonstrate the versatility of the approach by applying it to three diverse large deviation problems:

(1) small random perturbations of dynamical systems with continuous statistics,
(2) small random perturbations of dynamical systems with discontinuous statistics,
(3) the empirical measures of Markov chains with continuous statistics and with discontinuous statistics.

While our main goal is to exhibit a general methodology, the technique allows, in the examples considered, a weakening of the assumptions that have previously been used in proving the large deviation principle. We also obtain a number of new results.

Klaus Fleischmann (joint work with Ingemar Haj)

Large deviation probabilities for some rescaled superprocesses

Large deviations are discussed for the continuous super-Brownian motion in $\mathbb{R}^d$ in the case of an asymptotically small branching rate. Based on a complete blow-up property for the related cumulant equation some L^2-formula for the rate functional is derived. This formula might have some applications, as well as might give some hints concerning on eventual general theory for large deviations for measure-valued diffusions behind this particular example of a super-Brownian motion.

Mark Freidlin

Random perturbations of dynamical systems with conservation laws

The evolution of first integrals along the trajectories of the perturbed system is considered. After proper rescaling of time the first integral converges to a diffusion process on a graph corresponding to the conservation Law. Under certain assumptions concerning the non-perturbed system on the level set of the first integral the limiting process turns out to be Markovian. The limiting process is defined by a family of second order differential operators and by a collection of gluing conditions in the vertices. The operators are the result of averaging over the connected components of the level sets. The gluing conditions are calculated on the vertices of the graph corresponding to the saddle points of the first integral (if it is defined by a smooth function). The extremal points of the first integral correspond to those vertices which are inaccessible for the limiting process, and no boundary conditions should be given at these points.

Tadahisa Funaki

Hydrodynamic limit for one-dimensional exclusion processes

We consider a particle system on the one-dimensional periodic lattice with hard core exclusion. The jump rate is spatially homogeneous, non-degenerate and satisfies the detailed balance condition with respect to a trivial Hamiltonian $H \equiv 0$. The Bernoulli measures are therefore reversible for the dynamics. For this model, the non-equilibrium fluctuation problem (in the gradient case by using the method of Chang-Yau) and the hydrodynamic limit (in the general non-gradient case by applying the method of Varadhan; this part is due to Uchiyama) are discussed. The basic tools are the logarithmic Sobolev inequality and the spectral gap for the exclusion process.

Hans-Otto Georgii (joint work (in part) with H. Zessin)

Large deviations for Gibbsian point random fields

We present a large deviation principle for the stationary empirical fields for systems of marked point particles in boxes $\Lambda_n \uparrow \mathbb{R}^d$. The particle distributions are Gibbsian relative to one of the following types of interaction:

(1) interactions of possibly infinite range with hard-core repulsion,
(2) superstable pair interactions of finite range,
(3) interactions of mean-field type depending on the particle marks,
(4) nearest-particle interactions for $d = 1$.

In the cases (2) and (4) we impose periodic boundary conditions. Since the underlying topology is chosen fine enough, the contraction principle then gives us a large deviation principle for the "individual empirical fields" defined by averaging over the particle positions. We also present a maximum entropy principle implying a general version of the equivalence of ensembles.

Andreas Greven (joint work with Frank den Hollander)

A variational characterization of the speed of a one-dimensional self-repellent random walk

Let Q_n^α denote the probability measure of an n-step random walk $(0, S_1, \ldots, S_n)$ on $\mathbb{Z}$ obtained by weighting the simple random walk with the factor $(1-\alpha)$ for every self-intersection. This is a model for a one-dimensional polymer. We prove that for every $\alpha \in (0,1)$ there exists $\theta^*(\alpha) \in (0,1)$ such that

$$\lim_{n\to\infty} Q_n^\alpha\Big(\frac{|S_n|}{n} \in [\theta^*(\alpha)-\varepsilon,\, \theta^*(\alpha)+\varepsilon]\Big) = 1 \quad \text{for every } \varepsilon > 0.$$

We give a characterization of $\theta^*(\alpha)$ in terms of the largest eigenvalue of a one-parameter family of $\mathbb{N}\times\mathbb{N}$ matrices, which allows us to prove that θ^* is an analytic function, $\theta^*(0) = 0$, $\theta^*(1) = 1$, and $\theta^*(x) \in (0,1)$ for $x \in (0,1)$. Besides for the speed we prove a limit law for the local times of the walk. The techniques used enable us to treat more general forms of self-repellence involving multiple intersections.

C. Kipnis and S. Olla (joint work with C. Landim)

Hydrodynamics for the generalized exclusion process

The generalized exclusion process with at most two particles per site is one of the simplest infinite particle systems which is non-gradient with product-form invariant measures and for which one can prove hydrodynamical limits. The limiting equation is, as expected, of the form $\partial_t \varrho = \partial_x(\hat{a}(\varrho)\partial_x \varrho)$ where $\hat{a}$ is given by a variational formula.

A. I. Kometch (joint work with E. Kopylova and N. Ratanov)

The stabilization of statistics in wave equations with mixing

There exist many statistical equilibrium phenomena in physics related to Hamiltonian infinite-dimensional systems of mathematical physics, for example Gibbs measures in statistical mechanics and the black-body emission law in electrodynamics. The phenomena lead us to a problem of "statistical stabilization". This means that these statistics appear as $t \to \infty$ for the solutions of equations considered when the initial statistics at $t = 0$ is "almost arbitrary". We prove such stabilization for the linear wave equation and also for the Klein-Gordon equation with constant or variable coefficients in $\mathbb{R}^n$, where $n \geq 2$.

We assume that the initial statistics fit the Rosenblatt-Ibragimov mixing condition and that they are homogeneous in $x \in \mathbb{R}^n$. In the case of constant coefficients we use the explicit formula for the solution and apply the extention of the "rooms-corridors" method of S. N. Bernstein, M. Rosenblatt, and Ibragimov-Linnik. In the case of variable coefficients there is no of explicit formula. We reduce the case to the constant coefficients case by the scattering theory. But the total energy of solutions considered is infinity almost surely because of the homogeneity of the initial data (and the solutions). Then we must construct the scattering theory for solutions of infinite energy.
The result is: the statistics of solutions at time t converge to some *Gaussian* measure as $t \to \infty$. This is the analogue of the central limit theorem for the Hamiltonian systems considered. Note that the Gibbs measures for our linear equations "must" be Gaussian, because their Hamilton functions are quadratic forms. This means for the large deviations of the solutions, considered in each bounded region of space $\mathbb{R}^n$: We can almost surely tame the initial data to be very small bounded functions in $\mathbb{R}^n$. But, as $t \to \infty$, the solution at the considered point (or the energy in the considered region) may be arbitrary large.

A. P. Korostelëv

Action functional for dynamical systems with discontinuities

A well-known "continuous mapping" method is applied to a piecewise smooth dynamical system having a surface of "stable discontinuity". For such a system disturbed by a standard white Gaussian noise of a small intensity ε, i. e. for the solution of the stochastic equation

$$\dot{X}^\varepsilon(t) = b(X^\varepsilon(t)) + \varepsilon \dot{W}(t), \quad 0 \le t \le T,\ \varepsilon \to 0,\ X^\varepsilon(0) = 0,$$

the action functional (i. e. the rate function governing the large deviations) is obtained. The basic idea is that there exists a continuous mapping $F : C_{0,T} \to C_{0,T}$, which is Lipschitz in the space of continuous functions $C_{0,T}$ and satisfies $X^\varepsilon = F(\varepsilon W)$. Moreover, there exists another mapping $G \colon C_{0,T} \to C_{0,T}$ such that $G(\varepsilon W) = \pi^\varepsilon$ where $\pi^\varepsilon(t) = \int_0^t I(X_1^\varepsilon(s) > 0)\, ds$, i. e. $\pi^\varepsilon(t)$ is the staying-time of $X^\varepsilon(t)$ in the positive half-space (we assume without loss of generality that the surface of discontinuity is described by $x_1 = 0$). If $\varphi \in C_{0,T}$, and $\psi = F(\varphi)$, $\mu = G(\varphi)$, then the inverse mapping has an explicit expression: $\varphi = \psi - \int b_+(\psi)\, d\mu - \int b_-(\psi)\, d(t - \mu)$ where $b_\pm$ are one-sided limits of b on the surface of discontinuity. Thus, appling the large deviation principle for the Wiener process one gets without any cumbersome calculation the action functional for the joint process $(X^\varepsilon, \pi^\varepsilon)$:

$$I(\psi, \mu) = \frac{1}{2} \int_0^T \|\dot{\psi} - b_+ \dot{\mu} - b_-(1 - \dot{\mu})\|^2.$$

The extensions to jumping processes are discussed. It is known that if one of the three staying-times (in the positive or the negative half-space, or that on the surface of discontinuity) is vanishing, then the approach applies. In particular, the large deviations for the solution of $\dot{X}^\varepsilon = -c \operatorname{sgn}(X^\varepsilon) + \dot{\xi}^\varepsilon$, where ξ^ε is the rescaled Poisson process, are governed by the action functional

$$I(\psi, \mu) = \int_0^T L_0(\dot{\psi} + c\dot{\mu}) \quad \text{where } L_0(u) = 1 + u \log(u/e).$$

But the same equation noised by the two-sided Poisson process (jumps ± 1 with probability $1/2$) leads to a problem that has no simple solution.

C. Landim

An application of large deviation principles for the empirical measures of interacting particle systems

We consider the symmetric simple exclusion process for which a large deviation principle for the empirical measure was proved by Kipnis, Olla and Varadhan in finite volume and extended to infinite volume by Landim. We obtain a large deviation principle for the occupation time of a site in this model as a consequence of the previous result in one dimension.

Tzong-Yow Lee

Large deviations for branching diffusions

For a branching Brownian motion starting from the origin with multiplication rate $\varepsilon^{-1}C$ and diffusivity εD, write P^ε the for probability measure and E^ε for the expectation. We ask:

$$(1) \qquad P^\varepsilon \left\{ \begin{array}{l} \text{sample tree has at least one 1-branch in} \\ \text{a tiny "neighborhood" of } \varphi(s),\, 0 \le s \le 1 \end{array} \right\} \asymp ?\,,$$

$$(2) \qquad P^\varepsilon \left\{ \begin{array}{l} \text{sample tree has at least one 2-branch} \\ \text{in a tiny "neighborhood" of } (\varphi_1, \varphi_2) \end{array} \right\} \asymp ?\,,$$

$$(3) \qquad P^\varepsilon \{ R_1 \sim b_1, R_2 \sim b_2 \} \asymp ?\,,$$

where $\asymp$ means logarithmic equivalence as $\varepsilon \searrow 0$ and R_t denotes the position of the rightmost particle at time t. Problems (1) and (2) are answered, for problem (3) a partial solution is given.

J. T. Lewis

Thermodynamical aspects of large deviations

The use in risk theory of intensive parameters analogous to the thermodynamic temperature (Martin-Löf 1986) prompts the question: Under what conditions does the machinery of equilibrium thermodynamics apply in the theory of large deviations? In joint work with Ch. Pfister (Lausanne), we examine the thermodynamic formalism of Ruelle (1965) and Lanford (1973) in the setting of probability measures on Banach spaces. We define a Lanford entropy function and a grand canonical pressure and give conditions for the equivalence of ensembles. Motivated by Gibbs' axiomatization of thermodynamics (Gross 1982), we define a Gibbs entropy function. We give conditions for the Lanford entropy function to exist and be a Gibbs entropy function; we examine the connection with the large deviation principle (cf. O'Brien and Vervaat 1990).

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[3] Lanford, O. E.: 1971 Battelle Lectures, LNP 20 (1973).
[4] Gross, L.: Saint Flour X–1980, LNM 929 (1982).
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Matthias Löwe

Large deviations for U-statistics

Let $(X_i)_{i\in\mathbb{N}}$ be a sequence of i. i. d. random variables taking values in some Polish probability space X with common law π. It is well-known that

$$U_n := \frac{1}{\binom{n}{m}} \sum_{1\le i_1<\cdots<i_m\le n} h(X_{i_1},\ldots,X_{i_m})$$

and

$$V_n := \frac{1}{n^m} \sum_{1\le i_1,\ldots,i_m\le n} h(X_{i_1},\ldots,X_{i_m})$$

are “good” estimators for $E_{\pi^m}(h)$, where $h: X^m \to \mathbb{R}^d$ is some integrable function. Under the condition that the moment generating function of h and every “diagonal” of h exist, we derive a large deviation principle for the distributions of U_n and V_n. In both of the cases the rate function is given by

$$I(y) = \inf\Big\{H(\varrho \,|\, \pi) \;\Big|\; \varrho \in \mathcal{M}_1(X),\; \int h \, d\varrho^m = y\Big\}$$

where $H(\,\cdot\,|\,\pi)$ denotes the usual entropy with respect to π. Our key tools are the contraction principle, Sanov's theorem and a graph-theoretic result about the factorization of complete hypergraphs due to Baranyai.

Peter Major

Phase transition in random external magnetic field – a conjecture

We discussed a one-dimensional long-range interaction model with a random external magnetic field. Our conjecture is that there is a phase transition in this model at low temperatures. This conjecture follows from a large deviation result about the distribution of the average spin in this model. We claim that the rate function appearing in this result is not convex in a certain region. Thus convexity is the cause of the phase transition, and its appearance is closely related to the long-range interaction of the model.

M. B. Maljutow

Large deviations in search for significant variables of a function

A function $f(x_1, \ldots, x_t)$ of a vast number of variables may be expressed in the form $g(x_{\lambda_1}, \ldots, x_{\lambda_s})$ where $\lambda_1, \ldots, \lambda_s$ is a sequence of unknown indices and s is small compared to t. Choosing the sequence $\overline{X}(i) = (x_1(i), \ldots, x_t(i))$, $i = 1, \ldots, N$, arbitrarily, we observe the values of a random variable Z_i which are related to the sequence of $Y_i = f(\overline{X}(i))$ via transition probabilities $T(Z_i|Y_i)$. Measurements are independent given the sequence $\overline{X}(1), \ldots, \overline{X}(N)$. The main quantity of interest is the minimal sample size N_l which guarantees the correct decision on $\bar{\lambda} = \lambda_1, \ldots, \lambda_s$ with probability of error not exceeding ε. The cases of static and sequential designs are investigated. In both cases the upper estimate is $N_l \le \text{const} \times \ln t$ when $t \to \infty$ and s is constant. Of special interest is the additive smooth model

$g(x_1,\dots,x_s)=\sum_{\alpha=1}^{s} g_\alpha(x_\alpha)$, disturbed by an additive noise. Under the condition of subgaussian tales of errors A. Korostelev proved the large deviation estimate for the rather unexpected statistic-inconsistent estimate of $S_\alpha^2=\int_{-1}^{1} g_\alpha^2(x)\,dx$, assuming that all functions g_α and their derivatives are bounded and $S_\alpha^2 \ge \Delta > 0$. This estimate is the base for obtaining the estimate for N_l mentioned above. For sequential design the simple lemma on large deviations for supermartingales allows us to obtain the same asymptotics in a more simple way. Some lower bounds for N_l are reviewed and cases, where estimates for N_l are precise, are mentioned.

A. A. Mogulskii (joint work with A. A. Borovkov)

Large deviation theorems for likelihood estimators

Let $a_1(\theta), a_2(\theta),\dots$ be i. i. d. random fields in $(C(\Theta),B)$, where $C(\Theta)$ is a linear space of continous functions $f(\theta)$, $\theta\in\Theta$, and Θ is a closed bounded subset of $\mathbb{R}^k$. We call a vector $\theta_n^+\in\Theta$ at which $A_n(\theta)=a_1(\theta)+\dots+a_n(\theta)$ attains its maximum a *maximum point of* $A_n(\theta)$:

$$A_n(\theta_n^+)=\max_{\theta\in\Theta} A_n(\theta).$$

The vector θ_n^+ is not uniquely defined. Therefore, we define “upper” and “lower” distributions of θ_n^+ by the formulae

$$P_+(\theta_n^+\in B)\equiv P\big(\max_{\theta\in B} A_n(\theta)\ge \max_{\theta\in\Theta\setminus B} A_n(\theta)\big)$$

and

$$P_-(\theta_n^+\in B)\equiv P\big(\max_{\theta\in B} A_n(\theta)> \max_{\theta\in\Theta\setminus B} A_n(\theta)\big).$$

In this talk we study the “fine” asymptotics of the sequence

$$P_\pm(\theta_n^+\in B).$$

Peter E. Ney

Large deviations in $\mathbb{R}^d$

Let $X_1,X_2,\dots$ be i. i. d. random variables taking values in $\mathbb{R}^d$, $S_n=\sum_1^n X_i$, $\Lambda(\alpha)=E\,e^{\langle\alpha,X_1\rangle}$ for $\alpha\in\mathbb{R}^d$, and $\mathcal{D}(\Lambda)=\{\,\alpha:\Lambda(\alpha)<\infty\,\}$. If $\mathcal{D}(\Lambda)$ does not contain a neighborhood of the origin, then the level sets of $\Lambda^*(x)=\sup_\alpha[\langle\alpha,x\rangle-\Lambda(\alpha)]$, $x\in\mathbb{R}^d$, will not be compact, and the large deviation principle upper bound may fail. However, if the level sets of Λ^* can be suitably approximated by half-spaces, then an upper bound can be proved. Necessary and sufficient conditions are given for such an approximation to be possible. They boil down to the property that the gererating functions of certain marginal random variables should not be degenerate (i. e. $\not\equiv \infty$ away from 0). The above results are extended to approximation and separation theorems for the conjugate f^* of an essentially arbitrary convex function f. The hypotheses are expressed in terms of the domain $\mathcal{D}(f)$. This leads to a classification of the sections of f^* into "elliptic", "parabolic" and "hyperbolic" classes, which are natural extensions of the conic sections.

Esa Nummelin

A matrix representation for the one-dimensional transfer operator

We consider the transfer operator $\mathcal{L}$ defined by

$$\mathcal{L}f(i_{-\infty}^{0}) = \sum_{i_1} l(i_{-\infty}^{1}) f(i_{-\infty}^{1})$$

where $i_{-\infty}^{0} \in A^{\times \mathbb{N}_-}$, A is a finite alphabet, l and f are lower semicontinuous non-negative functions on $A^{\times \mathbb{N}_-}$. We construct a non-negative matrix Q with index set S equal to the collection of finite sequences of A-symbols, and such that

$$\mathcal{L}I_{j^*} = \sum_{i^*} Q(i^*, j^*) I_{i^*},$$

where I_{i^*} is the indicator of a cylinder $i^* \in S$. Under the usual variation conditions we establish positive and geometric recurrence properties of Q. These are related to the eigenvalue problem for the transfer operator $\mathcal{L}$ (Ruelle's Perron-Frobenius theorem).

E. A. Pechersky

The large deviations for a simple information network

We consider a tandem system as on this picture

$$\longrightarrow \boxed{\quad 1 \quad} \longrightarrow \boxed{\quad 2 \quad} \longrightarrow$$

defined by i. i. d. vectors $(\tau_i, \xi_i^1, \xi_i^2)$. These τ_i are intervals between messages and the ξ_i^j are the times for transmitting messages through the j-th node. We assume that $P(\tau_i > x) = e^{-\lambda x}$, $E\xi_i^j = \mu_j$, and $\varphi_j(\theta) = Ee^{\theta\xi_1^j}$. Then $(1/x)\log P(\omega > x) \to -\min\{\beta_1, \beta_2\}$, where ω is the total waiting time of a message in the tandem, and the β_j are defined by the equation $\beta_j = \lambda[\varphi_j(\beta_j) - 1]$.

Ch. Pfister

Large deviations and the isoperimetric problem in Ising model

The rate function of the empirical magnetization is computed explicitly in the case of coexistence of phases. The rate function is given by the minimum of a variant of the classical isoperimetric problem. The computation is done in two dimensions. If $\tau(n)$ is the surface tension in the direction $n \in \mathbb{R}^2$, $\|n\| = 1$, then for $-m^* < x < m^*$

$$\lim_{L\to\infty} -\frac{1}{L} \ln \Pr_{\mu^+_{\Lambda_L}} \left(\frac{1}{|\Lambda_L|} \sum_{t\in\Lambda_L} \sigma(t) \sim x \right)$$
$$= \min\left\{ \int_\gamma \tau(n) : \gamma \text{ closed simple curve such that } \gamma \subset \Lambda_L \right.$$
$$\left. \text{and the interior of } \gamma \text{ has volume } \tfrac{m^* - x}{2m^*} \right\}$$

where $\mu^+_{\Lambda_L}$ is the Gibbs measure in a square box Λ_L of volume L^2, with (+)-boundary condition and $m^* = \lim_{L\to\infty} \mathbb{E}_{\mu^+_{\Lambda_L}}(\sigma(t))$.

A. A. Puhalskii

Weak convergence theory approach to large deviations

We use ideas and methods of weak convergence theory to establish large deviation results analogous to those for weak convergence. The main result is an analogue of Prohorov's theorem. Say that a sequence $(P_n)_{n\in\mathbb{N}}$ of probability measures on the Borel σ-field of a topological space is large deviation relatively compact if any subsequence contains a further subsequence obeying the large deviation principle with some rate function. Then the theorem states that for a Tychonov space exponential tightness of $(P_n)_{n\in\mathbb{N}}$ implies large deviation relative compactness. For a Polish space the converse is also true. The theorem is applied to study large deviations of semimartingales. To this end, we introduce for large deviations analogues of the methods of finite dimensional convergence and of martingale problems in weak convergence. This allows us to obtain new results on large deviations of semimartingales with paths in the Skorohod space.

Jeremy Quastel

Large deviations from a hydrodynamic scaling limit for a non-gradient system

We consider the symmetric simple exclusion process with coloured, but mechanically identical particles as a simple, but physically motivated, example of a non-gradient system. Coulour density profiles are shown to have a hydrodynamic scaling limit which appears as a law of large numbers for an appropriate sequence of measures. The limiting equation has the form

$$\frac{\partial \vec{\varrho}}{\partial t} = \frac{1}{2}\nabla A(\vec{\varrho})\chi(\vec{\varrho})\nabla\vec{\varrho}, \qquad \vec{\varrho}(0) = \vec{\varrho}^{\,0},$$

where A is a matrix involving the self-diffusion constant, $D_s(p)$ is the limiting covariance of a test particle in density ϱ, and χ denotes the compressibility. Large deviations are calculated from this scaling limit with a rate function which is approximately the H_{-1} norm with "weights" $A^{-1}(\varrho)$.

Uwe Schmock

Maximun entropy principle for uniformly ergodic Markov chains

Results of Bolthausen and myself (1989) about the maximum entropy principle for the empirical process of uniformly ergodic discrete-time Markov chains are extended to more general empirical processes by putting more restrictive assumptions on the functional H of the empirical process $\{L_n\}_{n\in\mathbb{N}}$. Using a special construction, multivariate empirical processes and certain continuous-time Markov processes with continuous paths can be treated. The weak accumulation points of the sequence $\{\mathbb{P}_n\}_{n\in\mathbb{N}}$ of transformed path measures, defined by

$$\widehat{\mathbb{P}}_n(A) = \frac{\mathbb{E}[1_A \exp(nH(L_n))]}{\mathbb{E}[\exp(nH(L_n))]}, \qquad A \subset \Omega \text{ measurable},$$

are mixtures of Markov chains minimizing a certain free energy. The proof relies on large deviation results in the τ-topology for Markov process, which are due to Bolthausen (1987).

S. B. Shlosman

Droplet condensation: Large and moderate deviations at the phase transition

Deviations are studied for the sum $S_N = \sigma_1 + \cdots + \sigma_N$ of the random variables $\sigma_i = \pm 1$, which are distributed according to the Ising model random field with inverse temperature $\beta \gg \beta_{\text{crit}}$, on the ν-dimensional lattice. The magnetic field is zero, and (+)-phase is considered. It is shown that for deviations b such that $b - E(S_N) \geq -c\,N^\alpha$ with $\alpha < \nu/(\nu+1)$, for some $c > 0$, one has

$$\Pr\{S_N = b\} = \frac{2}{\sqrt{2\pi D_{N,b}}} \exp\{-I_N(b)\}(1 + o_N(1)),$$

where $D_{N,b}$ is the "tilted" variance, and $I_N(\,\cdot\,)$ is the rate function. In the complementary region $b - E(S_N) \leq -c\,N^\gamma$, $\gamma > \nu/(\nu+1)$, $c > 0$, one has

$$\frac{\ln \Pr\{S_N = b\}}{(E(S_N) - b)^{(\nu-1)/\nu}} = O_N(1).$$

So the region of deviations around $E(S_N) - N^{\nu/(\nu+1)}$ contains a threshold where the condensation of microscopic ($\sim \ln N$) droplets to macroscopic droplet ($\sim N^\kappa$ with $\kappa \geq 1/(\nu+1)$) takes place.

Herbert Spohn

Large scale dynamics in stochastic models for interfaces

The statistical mechanics of surfaces is modelled conveniently in terms of effective interface models. They are given by a real valued field, ϕ, over the lattice $\mathbb{Z}^d$. The surface is the graph of this function. The field has the energy

$$H = \sum_{\langle x,y \rangle} V(\phi(x) - \phi(y)),$$

where $\langle x, y\rangle$ is a pair of nearest neighbors and V is convex and bounded as $V(\phi) \geq c|\phi|^{1+\delta}$, $\delta > 0$. Clearly H is invariant under the global shift $\phi(x) \to \phi(x) + a$, which is needed to have the interpretation of a surface energy. To H there corresponds a d-parameter family of Gibbs measures. They should be thought of being defined on the difference variables $\phi(y) - \phi(x)$, $|x - y| = 1$. They are defined by taking the infinite volume limit at fixed tilt, $\phi(x) = u \cdot x$ for $x \in \partial\Lambda$. We consider pure relaxational dynamics

$$d\phi_t(x) = \frac{\partial H}{\partial \phi(x)}(\phi_t)\,dt + dW_t(x)$$

with independent Brownian motions of each site. The goal is to prove a law of large numbers in the form

$$\lim_{\varepsilon \to 0} \varepsilon^d \sum_x f(\varepsilon x)\varepsilon\phi_{\varepsilon^{-2}t}(x) = \int d^d r\, f(r) h(r,t). \qquad (*)$$

The macroscopic height profile should satisfy

$$\frac{\partial}{\partial t} h_t = \mu \sum_{\alpha=1}^{d} \frac{\partial}{\partial r_\alpha} \sigma_\alpha(\nabla h_t)$$

with μ the mobility and σ the surface tension, $\sigma_\alpha(u) = \frac{\partial}{\partial u_\alpha}\sigma(u)$. Elements of the proof of $(*)$ are discussed.

Josef Steinebach

Exponential large deviations of the mean under special spherical distributions

Consider an n-dimensional sample $X = (X_1, \ldots, X_n)$ under a spherical distribution, i. e. $X = \mu + e$, $\mu \in \mathbb{R}^n$, where the distribution of the error vector e has a λ^n-density

$$f(x; g) = c(n; g) g(\|x\|^2), \quad x \in \mathbb{R}^n, \tag{1}$$

generated by a nonnegative measurable function g with positive normalization $c(n; g)$. We are interested in convergence rates of the least squares estimate of a possible common mean of $X_1, \ldots, X_n$, that is, we want to investigate the large deviations of

$$P(A_n) = P(|\bar{X}_n - \bar{\mu}.| > \varepsilon) = 2P\biggl(\sum_{i=1}^{n}(X_i - \mu_i) > \varepsilon\biggr),$$

where $\varepsilon > 0$, $\bar{X}_n = (1/n)\sum_{i=1}^n X_i$ and $\bar{\mu}. = (1/n)\sum_{i=1}^n \mu_i$. For a class of spherical distributions generated by a function g of type

$$g(r^2) = a\, r^b e^{-cr^d}, \quad r > 0, \tag{2}$$

(a, b, c, d positive constants), the following large deviation results can be established:

Theorem. Under spherical distribution of X according to (1) with g as in (2), we have as $n \to \infty$,

(i) if $d \geq 2$, then $\log P(A_n) \sim -c(n\varepsilon^2)^{d/2}$,
(ii) if $1 < d \leq 2$, then $\log P(A_n) \sim -\frac{1}{2}(cd)^{2/d} n^{2(1-1/d)} \varepsilon^2$,
(iii) if $0 < d \leq 1$, then $P(A_n)/\exp(-\alpha n^\beta) \to +\infty$ for all $\alpha, \beta > 0$ ("no exponential rate").

Alain-Sol Sznitman

Brownian motion in a Poissonian potential

I describe in this talk certain large deviation principles which govern the behavior of Brownian motion moving in a typical Poissonian potential. These large deviation principles involve the construction, via a shape theorem, very much in the spirit of first passage percolation, of certain constants which generalize the Lyapounov exponents in one dimension. These large deviation results then enable us to study Brownian motion with a constant drift h moving in the same potential and to describe the transition of regime which occurs between small h and large h.

Srinivasa R. S. Varadhan (joint work with S. Olla and H.-T. Yau)

Hydrodynamic limit for Hamiltonian systems with noise

We consider a Hamiltonian system of N particles in the phase space $(\mathbb{T}^3 \times \mathbb{R}^3)^N$ evolving under a short range pair potential of the form $V((x-y)/\varepsilon)$ where ε is a scale parameter related to N by $N\varepsilon^3 = 1$. We aim to establish a relationship between the Hamiltonian dynamics and the corresponding Euler equation derived by the thermodynamic formalism. In order to achieve this some small noise is added to the velocity components in such a way as not to destroy the conservation of momenta and energy. The classical Hamiltonian is replaced by one with bounded velocities. Then in a regime where the Euler equation has a smooth solution, we show that a suitably prepared local Gibbs family of densities on the phase space constructed from the solutions of Euler equation is close to the corresponding time solution of the Hamiltonian system with noise.

Kongming Wang (joint work with J.-D. Deuschel)

Large deviations for the occupation time functional of a Poisson system of independent Brownian particles

Let $\{N_s\}_{s\geq 0}$ be the evolution system starting from N_0, a Poisson point process with intensity dx, where each particles independently follows the law of a d-dimensional Brownian motion. Take $\varphi \in L^1(\mathbb{R}^d)$ with compact support, and let $N_s(\varphi) = \sum_{x\in\text{supp}(N_0)} \varphi(B_s^x)$ and $L_T(\varphi)(t) = \int_0^t N_{Ts}(\varphi)\, ds$. We study the large deviations and central limit theorems for $L_T(\varphi)(t)$, $t \in [0,1]$. In the lower (recurrent) dimensions $d = 1, 2$ we have critical orders $T^{1/2}$ and $T/\log T$, whereas in higher (transient) dimensions we have the usual order T. We give explicit expressions for the corresponding rate functions and covariance functionals and derive some asymptotic microcanonical distributions.

W. A. Woyczyński (joint work with J. Szulga and J. A. Mann)

Large deviation techniques in analysis of monomolecular layers

The partition function of a statistical mechanical system of hard oval shaped molecules moving on real line and with rotational degree of freedom, is replaced by its Poissonized version, which, in turn, can be analyzed via large deviation techniques when considered in the thermodynamic limit.

H.-T. Yau (joint work with Shenglin Lu)

Spectral gap and logarithmic Sobolev inequality for the Glauber and Kawasaki dynamics

We prove that there is a spectral gap uniformly with respect to the volume and boundary condition for the Glauber dynamics. If the Glauber dynamics is replaced by the Kawasaki dynamics then the spectral gap is proved to shrink by $1/L^2$. We assume some mixing conditions for the Gibbs state to hold. Furthermore, we prove a similar result for the logarithmic Sobolev inequality except for the Kawasaki dynamics for dimension $d > 1$.

Sandy L. Zabell (joint work with I. H. Dinwoodie)

Large deviations for sequences of mixtures

Say that a family $\{ P_\theta^n : \theta \in \Theta, n \geq 1 \}$ is *exponentially continuous* if when $\theta_n \to \theta$, one has that $\{P_{\theta_n}^n\}$ satisfies a large deviation principle with rate function $\lambda(\theta, v)$ for each $\theta \in \Theta$. In this case, if μ is a measure on Θ, then $P^n := \int_\Theta P_\theta^n \, d\mu$ satisfies a large deviation principle with rate function $\inf\{ \lambda(\theta, v) : \theta \in \mathrm{supp}(\mu) \}$ provided Θ is compact and given weak regularity conditions; see Dinwoodie and Zabell, *Annals of Probability,* 1992. In this talk I discuss to what extent the conditions of this theorem can be weakened; a necessary and sufficient condition for exponential continuity is given; and a relationship with epiconvergence is discussed.

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