Motivation: Bernoulli Model for Defaults

- Bernoulli loss indicators
  \[ N_i = \begin{cases} 
  1 & \text{if obligor } i \text{ defaults (within one year),} \\
  0 & \text{otherwise.} 
\end{cases} \]

- Default probability \( p_i = \mathbb{P}(N_i = 1) \) for \( i = 1, \ldots, m \).
- Random number of defaults \( N = N_1 + \cdots + N_m \).
- Probability distribution for \( n \in \{0, \ldots, m\} \)
  \[ 
  \mathbb{P}(N = n) = \sum_{I \subseteq \{1, \ldots, m\}} \frac{\prod_{i \in I} p_i}{|I|} \prod_{i \notin I} (1-p_i) 
  \]

\( m = 1000, n = 100 \implies \binom{1000}{100} \approx 6.4 \times 10^{139} \) terms

Observations ...

- Already the Bernoulli model with independent loss indicators has far too many terms for the calculation of the portfolio loss distribution in the general case.

- In the general Bernoulli mixture model, individual terms are too complicated to compute numerically.

- Different exposures and recovery rates are not even considered.

... and Conclusions

- Simplifying assumptions are necessary.
- Approximations need to be considered.
Poisson Approximation
• \( X_1, \ldots, X_m \) independent default 0-1-indicators
• Intensity \( \lambda = \sum_{i=1}^{m} p_i \) with \( p_i = \mathbb{P}(X_i = 1) \)
• Number of default events \( W = \sum_{i=1}^{m} X_i \)
• Total variation distance
  \[
  d_{TV}(\mu, \nu) = \sup_{A \subset \mathbb{N}_0} |\mu(A) - \nu(A)|
  \]
Quality of Poisson approximation (Barbour/Hall, 1984):
  \[
  d_{TV}(\mathcal{L}(W), \text{Poisson}(\lambda)) \leq \frac{1 - e^{-\lambda}}{\lambda} \sum_{i=1}^{m} p_i^2
  \]
For full proof with Stein–Chen method, see e.g. Barbour, Holst and Janson: Poisson Approximation, Clarendon Press (1992).

Simple Poisson Model for Defaults
• Number \( N_i \) of defaults of obligor \( i \in \{1, \ldots, m\} \)
• Assume \( N_i \sim \text{Poisson}(\lambda_i) \) for all \( i \in \{1, \ldots, m\} \)
  (several defaults of an obligor possible).
• Assume independence of \( N_1, \ldots, N_m \).
• Random number of defaults \( N = N_1 + \cdots + N_m \).
• \( N \sim \text{Poisson}(\lambda) \) with \( \lambda = \lambda_1 + \cdots + \lambda_m \), i.e.,
  \[
  \mathbb{P}(N = n) = \frac{\lambda^n}{n!} e^{-\lambda} \quad \text{for all } n \in \mathbb{N}_0.
  \]
• \( m = 20, \lambda_i = 0.2 \implies \mathbb{P}(N > 20) \leq 2 \times 10^{-9} \).

Introduction to CreditRisk+, Standard Features
• Developed by Credit Suisse First Boston.
• Actuarial model for the aggregation of credit risks.
• Based on the Poisson approximation of individual defaults and the divisibility of the Poisson distribution.
• Allows for deterministic exposures/recovery rates.
• Several independent risk factors for dependence of default frequencies can be considered.
• Probability generating function \( \varphi_L \) of the credit portfolio loss \( L \) is available in closed form.
  → No Monte Carlo simulation, no stochastic error!

Extensions of CreditRisk+
• Stochastic losses of individual obligors are allowed, distribution may depend on the causing risk factor.
• Risk groups with dependent stochastic losses given default are possible.
• Risk factors for default frequencies may be dependent.
• Risk contributions of obligors can be calculated.
• Even with all the extensions, the probability generating function \( \varphi_L \) of the credit portfolio loss \( L \) is available in closed form.
  → No Monte Carlo simulation, no stochastic error!
• Distribution of \( L \) and risk contributions can be calculated from \( \varphi_{\gamma,L} \) with a numerically stable algorithm.
Input Parameters of CreditRisk$+^{}$ (Extended Version)

- Number of obligors $m \in \mathbb{N}$.
- Basic loss unit $E > 0$.
- Number $K \in \mathbb{N}_0$ of risk factors or non-idiosyncratic, (independent) default causes.
- Relative default variances $\sigma_k^2 > 0$ of risk factors $k \in \{1, \ldots, K\}$.
- Collection $G$ of nonempty subsets of all obligors $\{1, \ldots, m\}$, called risk groups.

Further Assumptions, Notation

- We assume that every obligor $i \in \{1, \ldots, m\}$ belongs to at least one group $g \in G$.
- Let $G_i := \{g \in G \mid i \in g\}$ denote the set of all risk groups to which obligor $i \in \{1, \ldots, m\}$ belongs, by assumption $G_i \neq \emptyset$.
- We assume that for each group the susceptibilities (also called weights) exhaustively describe the risk factors. That is, for all $g \in G$,
  $$\sum_{k=0}^{K} w_{g,k} = 1.$$
**Notation for Stochastic Losses**

Loss at default number \( n \in \mathbb{N} \) of risk group \( g \in G \) due to risk factor \( k \in \{1, \ldots, K\} \) or idiosyncratic risk \( k = 0 \)

- \( L_{g,i,k,n} \) part attributed to obligor \( i \in g \)
- \( L_{g,k,n} := \sum_{i \in g} L_{g,i,k,n} \) loss of entire group

Summation over default numbers, risks and groups:

- \( L_{g,k} := \sum_{n=1}^{N_{g,k}} L_{g,k,n} \) total loss of the group for risk \( k \)
- \( L_g := \sum_{k=0}^{K} L_{g,k} \) total of the risk group
- \( L := \sum_{g \in G} L_g \) portfolio loss

**Loss Attributed to Obligor \( i \in \{1, \ldots, m\} \)**

- Due to group \( g \in G_i \) and risk \( k \in \{0, \ldots, K\} \)
  \[ L_{g,i,k,n} := \sum_{n=1}^{N_{g,k}} L_{g,i,k,n} \]
- Due to risk \( k \in \{0, \ldots, K\} \)
  \[ L_{i,k} := \sum_{g \in G_i} L_{g,i,k} \]
- Total attributed loss
  \[ L_i := \sum_{k=0}^{K} L_{i,k} \]

**Probabilistic Assumptions for the Extended Version of CreditRisk+**

- For every group \( g \in G \) and every risk \( k \in \{0, \ldots, K\} \), the sequence of \( \mathbb{N}_0^g \)-valued random vectors \( (L_{g,i,k,n})_{i \in g} \)
  with \( n \in \mathbb{N} \) is i.i.d. and independent of all other random variables, with distribution
  \[ \mathbb{P}(L_{g,i,k,1} = \mu_i \text{ for all } i \in g) = q_{g,k,\mu}, \quad \mu \in \mathbb{N}_0^g. \]
- For each group \( g \in G \), the number \( N_{g,0} \) of idiosyncratic defaults is Poisson distributed according to the Poisson intensity \( \lambda_g \) and the susceptibility \( w_{g,0} \), i.e.,
  \[ N_{g,0} \sim \text{Poisson}(\lambda_g w_{g,0}) \quad \text{for every } g \in G. \]

**Probabilistic Assumptions (Cont.)**

- The group default numbers \( \{N_{g,0}\}_{g \in G} \) due to idiosyncratic risk are independent from one another and from all other random variables.
- The risks factors \( \Lambda_1, \ldots, \Lambda_K \) are independent, each one gamma distributed with \( \mathbb{E}[\Lambda_k] = 1 \) and \( \text{Var}(\Lambda_k) = \sigma_k^2 > 0 \), i.e., \( \alpha_k = \beta_k = 1/\sigma_k^2 \).
- For all groups \( g \in G \) and risks \( k \in \{1, \ldots, K\} \),
  \[ \mathcal{L}(N_{g,k}|\Lambda_1, \ldots, \Lambda_K) \overset{a.s.}{=} \mathcal{L}(N_{g,k}|\Lambda_k) \overset{a.s.}{=} \text{Poisson}(\lambda_g w_{g,k} \Lambda_k). \]
- Conditionally on \( \Lambda_1, \ldots, \Lambda_K \), the risk factor based defaults \( \{N_{g,k}|g \in G, \ k \in \{1, \ldots, K\}\} \) are independent.
Weighted Probability Generating Function

In order to calculate terms needed for the risk contributions we will need what we call weighted probability generating functions.

**Definition:** For $L : \Omega \rightarrow \mathbb{N}_0$ and an integrable random variable $X : \Omega \rightarrow \mathbb{R}$, we define the $X$-weighted probability generating function by

$$
\phi_{L,X}(s) = \mathbb{E}[X^s L] = \sum_{n=0}^{\infty} \mathbb{E}[X 1_{\{L=n\}}] s^n,
$$

which is meaningful at least for all $s \in \mathbb{C}$ with $|s| \leq 1$.

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Weighted Probability Generating Function (Cont.)

We will need expressions of the form $\mathbb{E}[\Lambda_k 1_{\{L=n\}}]$ for $k \in \{1, \ldots, K\}$ and $n \in \mathbb{N}_0$, which can be derived by

$$
\phi_{L,\Lambda_k}(0) = n! \mathbb{E}[\Lambda_k 1_{\{L=n\}}].
$$

Unifying approach for the $\gamma$-weighted probability generating function of the loss:

Fix $\gamma = (\gamma_1, \ldots, \gamma_K) \in [0, \infty)^K$ and define

$$
\phi_{L,\gamma}(s) := \mathbb{E}[\Lambda_{\gamma_1}^1 \cdots \Lambda_{\gamma_K}^K s^L], \quad |s| \leq 1,
$$

for the risk factors $\Lambda_{\gamma_1}, \ldots, \Lambda_{\gamma_K}$ and the total loss $L$.

$\gamma = 0$ gives the probability generating function $\phi_L$ of $L$.

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The Closed Form of the WPGF

$$
\phi_{L,\gamma}(s) = C_{\gamma} \exp\left(\bar{\lambda}_0(\phi_0(s) - 1) - \sum_{k=1}^{K} \left(\frac{1}{\sigma_k} + \gamma_k\right) \log(1 - \bar{\lambda}_k \sigma_k^2(\phi_k(s) - 1))\right),
$$

where $C_{\gamma} := \prod_{k=1}^{K} \mathbb{E}[\Lambda_{\gamma_k}^k] = 1$ if all $\gamma_k \in \{0, 1\}$, with PGF of mixture distributions (conditioned to be positive)

$$
\phi_k(s) := \sum_{g \in G} \frac{\lambda_g w_{g,k}}{\bar{\lambda}_k} \phi_{L,g,k,1}(s), \quad \bar{\lambda}_k := \sum_{g \in G} \lambda_g w_{g,k}(1 - q_{g,k,0}^s).
$$


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Weighted Probability Generating Function (Cont.)

**Definition:** A probability distribution $\{q_n\}_{n \in \mathbb{N}_0}$ is said to belong to the Panjer($a, b, k$) class with $a, b \in \mathbb{R}$ and $k \in \mathbb{N}_0$ if $q_0 = q_1 = \cdots = q_{k-1} = 0$ and

$$
q_n = \left(a + \frac{b}{n}\right) q_{n-1} \quad \text{for all } n \in \mathbb{N} \text{ with } n \geq k + 1.
$$

**Important Examples:** (all distributions are known)

- **Poisson($\lambda$) $\in$ Panjer(0, $\lambda$, 0) with $\lambda > 0$**
- **NegBin($\alpha$, $p$) $\in$ Panjer($q$, ($\alpha - 1$)q, 0)**
- **Log($q$) $\in$ Panjer($q$, $-q$, 1)** with $q \in (0, 1)$ and $q_n = -\frac{q^n}{n \log(1-q)}$ for all $n \in \mathbb{N}$
**Extended Panjer Recursion**

If $L(N) \in \text{Panjer}(a, b, k)$, independent of the i.i.d. $\mathbb{N}_0$-valued sequence $\{X_n\}_{n \in \mathbb{N}}$, and $aP(X_1 = 0) \neq 1$, then $S := X_1 + \cdots + X_N$ satisfies

$$P(S = 0) = \varphi_N(P(X_1 = 0))$$

with $\varphi_N$ probability generating function of $N$, and

$$P(S = n) = \frac{1}{1 - aP(X_1 = 0)} \left( P(S_k = n)P(N = k) + \sum_{j=1}^{n} \left( a + \frac{bj}{n} \right) P(X_1 = j)P(S = n - j) \right)$$

for all $n \in \mathbb{N}$, where $S_k = X_1 + \cdots + X_k$.

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**Application of Extended Panjer Recursion**

Remark: Recursion scheme is numerically stable for Poisson($\lambda$), NegBin($\alpha, p$), and Log($q$).

Observation: If $N \sim \text{Poisson}(\lambda)$, independent of the i.i.d. sequence $\{X_n\}_{n \in \mathbb{N}}$ with $X_1 \sim \text{Log}(q)$, then

$$S = X_1 + \cdots + X_N \sim \text{NegBin}\left( \frac{-\lambda \log(1 - q)}{\log(1 - q)}, 1 - q \right).$$

Application: Calculate
- Mixture distribution $\varphi_k$ for risks $k \in \{0, \ldots, K\}$.
- Panjer recursion for log. dist. for risks $k \in \{1, \ldots, K\}$.
- Mixture distribution of $\varphi_0$ and recursion results.
- Final Panjer recursion for Poisson distribution.
Extended Logarithmic Distribution

For \( k \in \mathbb{N} \setminus \{1\} \) and \( q \in (0, 1] \) define \( q_0 = \ldots = q_{k-1} = 0 \),
\[
q_n = \frac{(n)_k^{-1} q^n}{\sum_{i=k}^{\infty} (i)_k^{-1} q^i}
\]
for \( n \geq k \).

ExtLog(\( k, q \)) is in Panjer(\( q, -kq, k \)).

Extended Negative Binomial Distribution

For \( k \in \mathbb{N} \), \( \alpha \in (-k, -k - 1) \) and \( p \in (0, 1) \) define
\[
q = 1 - p, q_0 = \ldots = q_{k-1} = 0 \text{ and }
q_n = \frac{(\alpha+n-1)_n q^n}{p^{-\alpha} - \sum_{j=0}^{k-1} (\alpha+j-1)_j q^j}
\]
for \( n \geq k \).

ExtNegBin(\( \alpha, k, p \)) is in Panjer(\( q, (\alpha - 1)q, k \)).

Example for Numerical Instability

Take \( N \sim \text{ExtNegBin}(\alpha, k, p) \) with \( k \in \mathbb{N} \), \( \varepsilon, p \in (0, 1) \) and \( \alpha = -k + \varepsilon \). Consider the loss distribution
\[
\mathbb{P}(X_1 = 1) = \mathbb{P}(X_1 = l) = 1/2 \text{ with } l \geq 3. Then}
\[
p_{k+l} = q^{k(l-1)+\varepsilon k} \frac{q_k}{k+l} \left( \frac{q_k}{2^{k+l}} + \frac{q_{k+l+1}}{k+1} \right) - q^{k(l-1) - \varepsilon l} \frac{k}{k+l} \frac{q_k}{2^{k+l}}.
\]

With \( \varepsilon = 1/10000 \), \( k = 1 \), \( l = 5 \), \( p = 1/10 \):
\[
p_6 = 0.1499926 - 0.1499701 = 0.0000225.
\]

Panjer recursion with five significant digits gives
\[
p_6 = 0.0000400. \ldots
\]
Combination of Truncated Distributions

Fix $k \in \mathbb{N}_0$, $l \in \mathbb{N}$ and $\alpha_1, \ldots, \alpha_l, \beta_1, \ldots, \beta_l \geq 0$, not all zero. For $i \in \{1, \ldots, l\}$ assume that the $\mathbb{N}_0$-valued $\tilde{N}_i$ satisfies $\mathbb{P}(\tilde{N}_i < k + l - i) = 0$. Consider $q_0, \ldots, q_{k+l-1} \geq 0$ with $q_0 + \cdots + q_{k+l-1} \leq 1$. Define

$$q_n := c \sum_{i=1}^{l} \left( \alpha_i + \frac{\beta_i}{n} \right) \mathbb{P}(\tilde{N}_i = n - i) \quad \text{for } n \geq k + l,$$

$$c := \left(1 - \sum_{n=0}^{k+l-1} q_n\right) / \sum_{i=1}^{l} \left( \alpha_i + \frac{\beta_i}{i + \tilde{N}_i} \right).$$

Then $\{q_n\}_{n \in \mathbb{N}_0}$ is a probability distribution satisfying the recursion condition with $a_i = c\alpha_i$ and $b_i = c\beta_i$.

Weighted Convolution for ExtLog

Let $k \in \mathbb{N}$ and $q \in (0,1)$. Let $N \sim \text{ExtLog}(k+1, q)$ and $\tilde{N} \sim \text{ExtLog}(k, q)$, where $\text{ExtLog}(1, q)$ means $\text{Log}(q)$. Define $S = X_1 + \cdots + X_N$ and $\tilde{S} = X_1 + \cdots + X_{\tilde{N}}$. Then, with an explicit $b_1 > 0$, the weighted convolution

$$\mathbb{P}(S = n) = \frac{b_1}{n} \sum_{j=1}^{n} j \mathbb{P}(X_1 = j) \mathbb{P}(\tilde{S} = n - j), \quad n \in \mathbb{N},$$

is numerically stable. Algorithm:

- Panjer recursion for $\text{Log}(1, q)$
- $k-1$ weighted convolutions: $\text{Log}(1, q) \to \text{ExtLog}(2, q) \to \cdots \to \text{ExtLog}(k-1, q) \to \text{ExtLog}(k, q)$

Measuring Risk by Quantiles

Let $X$ be a loss variable and $\delta \in (0,1)$ a level.

**Definition:** Lower $\delta$-quantile of $X$

$$q_\delta(X) := \min\{x \in \mathbb{R} \mid \mathbb{P}(X \leq x) \geq \delta\}.$$ 

**Remark:** Quantiles are used as value-at-risk, they have bad properties concerning diversification.

**Properties:** $q_\delta(X)$ can jump when

- the level $\delta$ varies slightly,
- the loss variable $X$ varies slightly.
Measuring Risk by Expected Shortfall

Let \( X \) be a loss variable and \( \delta \in (0, 1) \) a level.

**Definition:** The expected shortfall is defined as

\[
\text{ES}_\delta[X] := \frac{E[X \mid X > q_\delta(X)] + q_\delta(X)(\mathbb{P}(X \leq q_\delta(X)) - \delta)}{1 - \delta}.
\]

**Remark:** If \( \mathbb{P}(X \leq q_\delta(X)) = \delta \), in particular if the distribution function \( \mathbb{R} \ni x \mapsto \mathbb{P}(X \leq x) \) of \( X \) is also left-continuous at \( x = q_\delta(X) \), then

\[
\text{ES}_\delta[X] = \mathbb{E}[X \mid X > q_\delta(X)].
\]

Calculation of Expected Shortfall in CreditRisk+

- Credit portfolio loss \( L \) is a discrete random variable, \( \to \) More complicated definition has to be used.
- The lower quantile \( q_\delta(L) \) and \( \mathbb{P}(L \leq q_\delta(L)) \) can be calculated using the CreditRisk+ algorithm.
- Furthermore \( \mathbb{E}[L1_{\{L > q_\delta(L)\}}] = \mathbb{E}[L] - \mathbb{E}[L1_{\{L \leq q_\delta(L)\}}] \)

with

\[
\mathbb{E}[L] = \sum_{g \in \mathcal{G}} \sum_{k=0}^{K} \lambda_g w_{g,k} \mathbb{E}[L_{g,k,1}]
\]

and

\[
\mathbb{E}[L1_{\{L \leq q_\delta(L)\}}] = \sum_{l=1}^{\text{max}(L)} l \mathbb{P}(L = l).
\]

Contributions to Expected Shortfall – Definition

**Definition:** For a subportfolio loss \( X \in L_1(\mathbb{P}) \) within a portfolio loss \( Y \in L_1(\mathbb{P}) \) define the expected shortfall contribution at level \( \delta \in (0, 1) \) of \( X \) to \( Y \) by

\[
\text{ES}_\delta[X, Y] = \frac{E[X \mid Y > q_\delta(Y)] + \beta_Y E[X \mid Y = q_\delta(Y)]}{1 - \delta}
\]

where

\[
\beta_Y = \frac{\mathbb{P}(Y \leq q_\delta(Y)) - \delta}{\mathbb{P}(Y = q_\delta(Y))}
\]

if \( \mathbb{P}(Y = q_\delta(Y)) > 0 \) and 0 otherwise.

**Remark:** If \( \mathbb{P}(Y \leq q_\delta(Y)) = \delta \), then \( \beta_Y = 0 \) and

\[
\text{ES}_\delta[X, Y] = \mathbb{E}[X \mid Y > q_\delta(Y)].
\]

Contributions to Expected Shortfall – Calculation in Extended CreditRisk+

By consistency and linearity of the allocation

\[
\text{ES}_\delta[L] = \text{ES}_\delta[L, L] = \sum_{i=1}^{m} \sum_{g \in \mathcal{G}_i} \sum_{k=0}^{K} \text{ES}_\delta[L_{g,i,k}, L].
\]

Since

\[
\mathbb{E}[L_{g,i,k}1_{\{L > q_\delta(L)\}}] = \mathbb{E}[L_{g,i,k}] - \mathbb{E}[L_{g,i,k}1_{\{L \leq q_\delta(L)\}}],
\]

we will compute \( \mathbb{E}[L_{g,i,k}1_{\{L = l\}}] \) for \( l \in \{1, \ldots, q_\delta(L)\} \). This can be done using the following lemma.
Lemma on Risk Contributions in CreditRisk+

For every obligor \( i \in \{1, \ldots, m\} \), every group \( g \in G_i \) and total loss \( l \in \mathbb{N}_0 \),

\[
E[L_{g,i,0}1_{\{L=l\}}] = \lambda_g w_{g,0} \sum_{\nu=1}^{l} E[L_{g,i,0,1}\{L_{g,0,1}=\nu\}] P(L = l - \nu)
\]

and, for every risk \( k \in \{1, \ldots, K\} \),

\[
E[L_{g,i,k}1_{\{L=l\}}] = \lambda_g w_{g,k} \sum_{\nu=1}^{l} E[L_{g,i,k,1}\{L_{g,k,1}=\nu\}] E[\Lambda_k1_{\{L=l-\nu\}}].
\]

Possible Approaches to Operational Risk Modelling

The Basel committee defined three approaches towards the quantification of operational risk. The two simple ones define concrete formulae for the risk capital, namely

- Basic indicator approach (BIA).
- Standardized approach (SA).

To reduce supervisory capital needs, an individual

- Advanced measurement approach (AMA)

can be chosen.

Business Lines for Operational Risk

- Eight business lines in the standardized approach:
  
  1. Corporate finance,
  2. Trading & sales,
  3. Retail banking,
  4. Commercial banking,
  5. Payment & settlement,
  6. Agency services,
  7. Asset management,
  8. Retail brokerage.

- These business lines also serve as categories for an advanced measurement approach.

Seven Loss-Types Distinguished for the Advanced Measurement Approach

- Internal fraud
- External fraud
- Employment practices & workplace safety
- Clients, products & business practice
- Damage to physical assets
- Business disruption & system failures
- Execution, delivery & process management
Application of Extended CreditRisk\(^{+}\) Methodology to Operational Risk: Reinterpretation of the Credit Risk Notation

- Number \(m\) of obligors \(\rightarrow\) number of business lines (\(m = 8\) for the ones given in the Basel committee’s document is an appropriate choice).
- Basic loss unit \(E\) stays the same (\(E = 10000\)).
- Number \(K\) of non-ideosyncratic risk factors \(\rightarrow\) number of loss types (\(K = 7\) for above list).
- Numbers \(\sigma^{2}_{k} > 0\) denote the relative variance of occurrences of losses of type \(k \in \{1, \ldots, K\}\).
- \(G\) contains the subsets of all business lines which can incur a loss due to the same event.

Notation for Business Lines and Risk Groups

We need for every risk group \(g \in G\) of business lines

- the (one year) intensity \(\lambda_{g} \geq 0\) for being hit by an operational loss event,
- the conditional probability \(w_{g,0} \in [0,1]\) for an idiosyncratic operational loss event not to belong to the types in \(\{1, \ldots, K\}\), of course \(w_{g,0} = 0\) is a possible choice,
- the conditional probabilities \(w_{g,k} \in [0,1]\) for an operational loss event to be of type \(k \in \{1, \ldots, K\}\).

Operational Risk Management

With the adoption of the extended CreditRisk\(^{+}\) model for operational risk, a risk manager can

- calculate the distribution of the operational loss,
- calculate risk measures such as value-at-risk and expected shortfall (might be infinity)
- and identify risky business lines and groups by their risk contribution (in case of finite expected shortfall).