

On q -Optimal Signed Martingale Measures in Exponential Lévy Models

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Dynamic and Static Problem

Dynamic Problem: $V_\xi(\tilde{x}) = \sup_{Y \in \mathcal{W}_C(\tilde{x})} E[U(Y_T)]$



Static: $\sup_{X \in \Theta(2m, \tilde{x})} E[U(X)] \Leftrightarrow$ Dual: $\sup_{y \geq 0, Z \in \mathcal{D}_a^q} E[\check{U}(yZ_T) + \tilde{x}y]$

where

- ◊ $\mathcal{W}_C(\tilde{x}) = \{Y \mid Y_t = \tilde{x} + \int_0^t N dS - C_t, N \in \mathcal{A}^p, C \in \mathcal{K}^p\}$
- ◊ $\Theta(p), \tilde{x} = \{X \in L^p(\mathcal{F}_T), \forall Z \in \mathcal{D}_a^q E(Z_T X) \leq \tilde{x}\}, \tilde{x} \in \mathbb{R}.$

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Finding Optimal Solutions

dynamic optimization problem



constrained static problem \longrightarrow Lagrange functional \longrightarrow dual problem

optimal \uparrow solution

saddle \uparrow point



$$I(\lambda_{\tilde{x}}^*) := (U')^{-1}(\lambda_{\tilde{x}}^*) \longleftarrow (\lambda_{\tilde{x}}^*, I(Z_{\mathcal{Y}(\tilde{x})}\mathcal{Y}(\tilde{x}))) \longleftarrow \lambda_{\tilde{x}}^* = Z_{\mathcal{Y}(\tilde{x})}\mathcal{Y}(\tilde{x})$$

Finding Optimal Solutions

dynamic optimization problem



constrained static problem ← dual problem

↓ optimal solution ↑

$$I(\lambda_{\tilde{x}}^*) := (U')^{-1}(\lambda_{\tilde{x}}^*) \longrightarrow \lambda_{\tilde{x}}^* = Z_{\mathcal{Y}(\tilde{x})} \mathcal{Y}(\tilde{x})$$

Dual Optimizer via Verification

- 1 Observe: in many cases, the optimal dual optimizer $\hat{Z}_{\mathcal{Y}(\tilde{x})}$ is independent of \tilde{x} so set $\hat{Z} = \hat{Z}_{\mathcal{Y}(\tilde{x})}$.
 - 2 Propose candidate $\check{Z} \in \mathcal{D}$. ($\mathcal{Y}(\tilde{x})$ can be easily derived if $\hat{Z} = \hat{Z}_{\mathcal{Y}(\tilde{x})}$).
 - 3 $X_0(\tilde{x}) := I(\mathcal{Y}(\tilde{x})\check{Z}_T)$ is optimal solution of $\sup_X E(U(X))$, s.t. $E(\check{Z}_T X) \leq \tilde{x}$
 - 4 Find strategy to replicate $X_0(\tilde{x})$
 $\Rightarrow X_0(\tilde{x})$ is optimal terminal value of the dynamic problem
- $\Rightarrow X_0(\tilde{x})$ is optimal solution of $\sup_X E(U(X))$, s.t. $\forall Z \in \mathcal{D} : E(Z_T X) \leq \tilde{x}$
- \Rightarrow By the duality relation $\mathcal{Y}(\tilde{x})\hat{Z}_T := \mathcal{Y}(\tilde{x})\check{Z}_T$ is optimal dual solution.

Convergence to the primal and dual solution of the exponential problem

$$\mathcal{Y}_{2m}(\tilde{x})Z_{2m} \longrightarrow \mathcal{Y}_{exp}(\tilde{x})Z_{min} \quad (\text{convergence of dual solutions})$$

$$\cong \cong$$

$$X_0^{(2m)}(\tilde{x}) \longrightarrow X_0^{(exp)}(\tilde{x}) \quad (\text{convergence of terminal wealths})$$

$$\cong \cong$$

$$V_{2m}(\tilde{x}) \longrightarrow V_{exp}(\tilde{x}) \quad (\text{convergence of value functions})$$

$$\cong \cong$$

$$\phi_{2m}(\mathcal{Y}_{2m}(\tilde{x})) \longrightarrow \phi_{exp}(\mathcal{Y}_{exp}(\tilde{x})) \quad (\text{convergence of the dual functions})$$

$$\cong \cong$$

$$\vartheta^{(2m)} \longrightarrow \vartheta^{exp} \quad (\text{convergence of portfolios})$$

Market Model

Let (Ω, \mathcal{F}, P) be a probability space, $T \in (0, \infty)$ a finite time horizon, and $\mathbb{F} = (\mathcal{F}_t)_{t \in [0, T]}$ a filtration satisfying the usual conditions, i.e. right-continuity and completeness.

We suppose that a discounted market with n assets is given by

$$S_t = \text{diag}(S_0^{(1)}, \dots, S_0^{(n)}) e^{\check{X}_t}, \quad t \in [0, T], \quad S_0^{(i)} > 0, \quad (1)$$

where \check{X} is supposed to be an \mathbb{R}^n -valued Lévy process with characteristic triplet $(\sigma\sigma^*, \nu, b)$ on (Ω, \mathcal{F}, P) and N a Poisson random measure with intensity measure $\nu(dx)dt$, and $\tilde{N}(dx, dt) = N(dx, dt) - \nu(dx)dt$ appearing in the Lévy-Itô-decomposition.

Market Model: Semimartingale Decomposition

We assume that the filtration \mathbb{F} coincides with $\mathbb{F}^{\check{X}}$, the completion of the filtration generated by the Lévy process \check{X} and $E[|S(t)|] < \infty$ for all $t \in [0, T]$. The second assumption guarantees that S is a special semimartingale with decomposition $S_t = S_0 + M_t + A_t$, where

$$dM_t = \mathbf{S}_{t-}(\sigma dW_t + \int_{\mathbb{R}_0^n} (e^x - \mathbf{1})\tilde{N}(dx, dt))$$

and

$$dA_t = \mathbf{S}_{t-}(-\beta + \int_{\mathbb{R}_0^n} (e^x - \mathbf{1} - x\mathbf{1}_{\|x\| \leq 1})\nu(dx))dt.$$

Here, $\beta = -(b + \frac{1}{2} \sum_j \sigma_{\cdot j}^2)$ and $\mathbf{S} = \text{diag}(S^{(1)}, \dots, S^{(n)})$. $\mathbf{1}$ denotes the vector in \mathbb{R}^n having all entries equal to one, and expressions such as e^x are to be interpreted componentwise, i.e. $e^x = (e^{x_1}, \dots, e^{x_n})'$.

Minimal Entropy Martingale Measure

We set,

$$\mathcal{D}_S^q = \{Z \in \mathcal{U}^q \mid E(Z_T) = 1, SZ \text{ is a local } P\text{-martingale}\},$$

where \mathcal{U}^q denotes the set of \mathbb{R} -valued $L^q(\Omega, P)$ -uniformly integrable martingales. A subset is $\mathcal{D}_a^q = \{Z \in \mathcal{D}_S^q \mid Z_T \geq 0 \text{ } P\text{-a.s.}\}$. We recall the definition of the minimal entropy martingale measure:

(Min_{e,log}) Find $Z^{\min} \in \mathcal{D}_a^{\log}$ such that

$$E[Z_T^{\min} \log Z_T^{\min}] = \inf_{Z \in \mathcal{D}_a^{\log}} E[Z_T \log Z_T].$$

where

$$\mathcal{D}_a^{\log} = \{Z \in \mathcal{D}_a^1, E(Z_T \log Z_T) < \infty\}.$$

$dQ_{\min} = Z_T^{\min} dP$ is called the *minimal entropy martingale measure* Q_{\min} (MEMM).

q -Optimal Martingale Measures

We further study the q -optimal signed martingale measure:

(Min $_{S,q}$) Find $Z^{(q)} \in \mathcal{D}_S^q$ such that

$$E[|Z_T^{(q)}|^q] = \inf_{Z \in \mathcal{D}_S^q} E[|Z_T|^q].$$

$dQ^{(S,q)} = Z_T^{(q)} dP$ is called the q -optimal signed martingale measure $Q^{(S,q)}$ (qSMM).

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Replace, \mathcal{D}_S^q by $\mathcal{D}_e^q = \{Z \in \mathcal{D}_S^q \mid Z_T > 0 \text{ } P\text{-a.s.}\}$, then the solution, provided its existence, defines the q -optimal equivalent martingale measure $Q^{(e,q)}$ (qEMM) with density process $\tilde{Z}^{(q)}$.

Assumptions

Assumption (C_q)

C_q^- : There exists an $\theta_q \in \mathbb{R}^n$ such that

$$\text{eg}_q(x) := ((q-1)\theta'_q(e^x - \mathbf{1}) + 1)^{\frac{1}{q-1}}$$

defines a real-valued function on the support on ν which satisfies

$$\sigma\sigma'\theta_q + \int_{\mathbb{R}_0^n} (e^x - \mathbf{1})\text{eg}_q(x) - x1_{\|x\| \leq 1} \nu(dx) = \beta \quad (2)$$

and

$$\int_{\mathbb{R}_0^n} |(\text{eg}_q^q(x)) - 1 - q(\text{eg}_q(x) - 1)| \nu(dx) < \infty. \quad (3)$$

C_q^+ : $(q-1)\theta'_q(e^y - \mathbf{1}) + 1 > 0, \nu$ -a.s.

If C_q^- and C_q^+ are satisfied, we say that C_q holds.

q -Optimal Equivalent Martingale Measure: Existence

Theorem (Jeanblanc et al., Theorem 2.9)

Suppose C_q holds. Then the q EMM exists and is given by

$$\mathcal{E}(\theta'_q \sigma, \text{eg}_q - 1),$$

where $\mathcal{E}(f, g)$ denotes the stochastic exponential with Girsanov parameters f, g , i.e.

$$\begin{aligned} \mathcal{E}_t(f, g) &= e^{\int_0^t f(s) dW_s - \frac{1}{2} \int_0^t \|f(s)\|^2 ds + \int_0^t \int_{\mathbb{R}^n} g(s, x) \tilde{N}(dx, ds)} \\ &\quad \times \prod_{s \leq t} (1 + g(s, \Delta \check{X}(s))) e^{-g(s, \Delta \check{X}(s))}. \end{aligned}$$

However, C_q^+ is very restrictive!

q -Optimal Equivalent Martingale Measure: Problems

Proposition

Suppose $n = 1$ and P is not a martingale measure. Then:

(i) If C_q holds for some $q > 1$, then

$$\int_{x \geq 1} e^{\theta e^x} \nu(dx) < \infty \quad (4)$$

for some $\theta > 0$ or the minimal entropy martingale measure does not exist.

(ii) If C_q holds for some $q > 1$, then

$$\int_{\mathbb{R}_0} (e^x - 1) - x 1_{|x| \leq 1} \nu(dx) + (b + \frac{1}{2} \sigma^2) < 0 \quad (5)$$

or upward jumps are bounded, i.e. $\nu([L, \infty)) = 0$ for some $L > 0$.

q -Optimal Equivalent Martingale Measure: Problems

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or upward jumps are bounded, i.e. $\nu([L, \infty)) = 0$ for some $L > 0$.

In particular, in a Kou or a Merton model C_q and the existence of the minimal entropy martingale measure cannot hold simultaneously. Condition (5) is rather unlikely (a negative optimal portfolio is induced). [▶ Examples](#)

q -Optimal Signed Martingale Measure: Existence

Theorem

Suppose that $q = \frac{2m}{2m-1}$ for some $m \in \mathbb{N}$ and that C_q^- holds. Then,

$$Z^{(q)} = \mathcal{E}(\theta'_q \sigma, \text{eg}_q - 1)$$

is the density process of q SMM.

Proposition

Suppose $n = 1$, $q(m) = \frac{2m}{2m-1}$, P is not a martingale measure, and the set of equivalent martingale measures is nonempty. Then, $C_{q(m)}^-$ holds for $m \in \mathbb{N}$, if and only if

$$\int_{x \geq 1} e^{2mx} \nu(dx) < \infty. \quad (6)$$

q -Optimal Signed Martingale Measure: Examples

Example

Suppose $n = 1$.

- (i) If $\nu(dx)$ behaves (up to a slowly varying function) as $e^{-\lambda_+x} dx$ for $x \rightarrow \infty$, then $C_{q(m)}^-$ holds for $m < \lambda_+/2$ and fails for $m > \lambda_+/2$. However, C_q fails for all q , if $\int_{\mathbb{R}_0} (e^x - 1) - x1_{|x| \leq 1} \nu(dx) + (b + \frac{1}{2}\sigma^2) > 0$. This tail behavior is inherent in generalized hyperbolic models and the Kou model.
- (ii) If there are constants $\eta_0, \eta_1 > 0$ such that

$$\int_{x \geq 1} e^{\eta_0 x^{1+\eta_1}} \nu(dx) < \infty, \quad (7)$$

then $C_{q(m)}^-$ holds for all $m \in \mathbb{N}$. However C_q fails for all q , if the upward jumps are not bounded and $\int_{\mathbb{R}_0} (e^x - 1) - x1_{|x| \leq 1} \nu(dx) + (b + \frac{1}{2}\sigma^2) > 0$. A popular model, which satisfies (7) and has unbounded upward jumps is the Merton model.

Minimal Entropy Martingale Measure

Assumption (C)

There exists a vector $\theta_e \in \mathbb{R}^n$ satisfying

$$\int_{\mathbb{R}_0^n} \|(e^x - \mathbf{1})e^{\theta_e'(e^x - \mathbf{1})} - x1_{\|x\| \leq 1}\| \nu(dx) < \infty \quad (8)$$

and $\theta_e' \sigma \sigma' + \int_{\mathbb{R}_0^n} (e^x - \mathbf{1})e^{\theta_e'(e^x - \mathbf{1})} - x1_{\|x\| \leq 1} \nu(dx) = \beta$.

Theorem (Fujiwara/Miyahara or Esche/Schweizer and Hubalek/Sgarra)

(i) If condition C is satisfied, then the entropy minimal martingale measure is given by

$$\mathcal{E}(\theta_e' \sigma, e^{\theta_e'(e^x - \mathbf{1})} - 1).$$

(ii) If $n = 1$ and there is no θ_e satisfying C, then the entropy minimal martingale measure does not exist.

Convergence to the Minimal Entropy Martingale Measure

Theorem (MEMM)

Suppose $n = 1$, the minimal entropy martingale measure exists, and there is a $\delta > 0$ such that θ_e , specified by condition C, satisfies

$$\int_{x \geq 1} e^{(\max\{\theta_e, -0.28\theta_e\} + \delta)e^x} \nu(dx) < \infty. \quad (9)$$

Then:

(i) If $\theta_e > 0$ or upwards jumps are bounded, then C_q is satisfied for sufficiently small $q > 1$ and the q -optimal equivalent martingale measures converge to the minimal entropy martingale measure in $L^r(P)$, for some $r > 1$, as $q \downarrow 1$ (in the sense that the densities converge).

(ii) Suppose $q(m) = \frac{2m}{2m-1}$. If $\theta_e < 0$, then $C_{q(m)}^-$ is satisfied for all $m \in \mathbb{N}$ and the $q(m)$ -optimal signed martingale measures converge to the minimal entropy martingale measure in $L^r(P)$, for some $r > 1$, as $m \uparrow \infty$.

Verification in a General Semimartingale Model

Theorem

Suppose $\hat{Z} \in \mathcal{D}_s^q$, $q = \frac{2m}{2m-1}$ and, for some $\tilde{x} < 2m$, the contingent claim

$$X^{(2m)}(\hat{Z}) := 2m - 2m\hat{Z}_T^{\frac{1}{2m-1}} \left(\frac{2m - \tilde{x}}{2mE(\hat{Z}_T^{\frac{2m}{2m-1}})} \right) \quad (10)$$

is replicable with a predictable strategy ϑ (#shares held) and $\vartheta \in \mathcal{A}^{2m}$, i.e.

$$\|\vartheta\|_{L^{2m}(M)} := \left\| \left(\int_0^T \vartheta d[M]_t \vartheta' \right)^{\frac{1}{2}} \right\|_{L^{2m}(\Omega, P)} < \infty, \quad (11)$$

$$\|\vartheta\|_{L^{2m}(A)} := \left\| \int_0^T |\vartheta dA_t| \right\|_{L^{2m}(\Omega, P)} < \infty. \quad (12)$$

Then \hat{Z} is the density process of the q -optimal signed martingale measure.

Replicating Strategy in the above Lévy Setting

Lemma

Suppose that $q = \frac{2m}{2m-1}$ for some $m \in \mathbb{N}$ and that C_q^- holds. Define

$$\begin{aligned} \vartheta_t^{(2m)} &= -\frac{2m - \tilde{x}}{2m - 1} \mathcal{E}_t((q - 1)\theta'_q \sigma, (q - 1)\theta'_q(e^{\cdot} - \mathbf{1})) \\ &\quad \times \theta'_q \mathbf{S}_{t-}^{-1} e^{t(q-1)\theta'_q(-\beta + \int_{\mathbb{R}_0^n} (e^x - \mathbf{1} - x \mathbf{1}_{\|x\| \leq 1}) \nu(dx))}. \end{aligned}$$

Then for $\tilde{x} \leq 2m$ and $\hat{Z} = \mathcal{E}(\theta'_q \sigma, e_{g_q} - 1)$ the contingent claim

$$X^{(2m)}(\hat{Z}) := 2m - 2m Z_q^{\frac{1}{2m-1}} \left(\frac{2m - \tilde{x}}{2m E(\hat{Z}_T^{\frac{2m}{2m-1}})} \right)$$

is replicable with initial wealth \tilde{x} and the predictable strategy $\vartheta^{(2m)} \in \mathcal{A}^{2m}$.

Consequences for Portfolio Management

The replicating strategy $\vartheta^{(2m)}$ is the solution of

$$\arg \max \{E(u_{2m}(X)); X \in \Theta^{(2m), \tilde{x}}\} \quad (13)$$

with respect to the utility function $u_{2m}(x) = -(1 - \frac{x}{2m})^{2m}$, where

$$\Theta^{(2m), \tilde{x}} = \left\{ X \in L^{2m}(\Omega, \mathcal{F}_T, P) : \exists \vartheta \in \mathcal{A}^{(2m)} \text{ s.t. } X = \tilde{x} + \int_0^T \vartheta_u dS_u \right\}.$$

Moreover under the assumptions of Theorem MEMM, $\vartheta^{(2m)}$ converges uniformly in probability to the optimal portfolio of the exponential problem, $U(x) = -e^{-x}$, $\vartheta^{(\infty)}$ and

$$\lim_{m \rightarrow \infty} \sup_{0 \leq t \leq T} \left| \left(x + \int_0^t \vartheta_u^{(2m)} dS_u \right) - \left(x + \int_0^t \vartheta_u^{(\infty)} dS_u \right) \right|$$

Note, if S is a one-dimensional (non-compensated) exponential Poisson process with jump height 2, there will be arbitrage but the portfolio problem (13) has a solution with $\theta_{q(m)} = -(2m - 1)!$

Conclusion

- 1 in the presence of jumps the q -optimal measure may fail to be equivalent, but belongs to the larger class of signed martingale measures
- 2 an analogous representation for the densities of equivalent martingale measures as stochastic exponentials is not available
⇒ techniques for the equivalent case cannot be generalized
- 3 a verification procedure based on a hedging problem yields an explicit representation of the q -optimal signed martingale measure
- 4 restrictive conditions for the equivalent case can be dropped
⇒ in many practically relevant models q MMM is signed
- 5 necessary and sufficient conditions for the existence of the q -optimal signed and equivalent measure are presented
- 6 convergence of the q -optimal measures to the minimal entropy martingale measure is established
- 7 consequences for the exponential utility problem are discussed

Some References (incomplete)

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q -Optimal Equivalent Martingale Measure: Examples ▶ back

(i) Note that most of the concrete models discussed in the literature, such as generalized hyperbolic models or the popular jump-diffusion models by Merton or Kou satisfy $\int_{x \geq 1} e^{\theta e^x} \nu(dx) = \infty$ for all $\theta > 0$. Hence, C_q and the existence of the MEMM cannot hold simultaneously for these models.

(ii) In condition (5) upward jumps are exponentially weighted and downward jumps are exponentially damped. Hence,

$$\int (e^x - 1) - x 1_{|x| \leq 1} \nu(dx)$$

can become negative only, if the Lévy measure gives much more weight to negative jumps than to positive jumps, leading to an extreme gain-loss asymmetry in the jumps. In such situation we expect that the deterministic trend b is large to compensate for the risk of downward jumps. So condition (5) may be rather unlikely to occur.

Proof

▸ Verification

Let $q = 2m/(2m - 1)$. We consider the following maximization problems with utility function $u_{2m}(x) = -(1 - \frac{x}{2m})^{2m}$:

$$\text{Max}_1 : X^{(1)} := \arg \max \{E(u_{2m}(X)); X \text{ s.t. } E(\hat{Z}_T X) \leq \tilde{x}\}$$

$$\text{Max}_2 : X^{(2)} := \arg \max \{E(u_{2m}(X)); X \text{ s.t. } \forall Z \in \mathcal{D}_S^q : E(Z_T X) \leq \tilde{x}\}$$

$$\text{Max}_3 : X^{(3)} := \arg \max \{E(u_{2m}(X)); X \in \Theta^{(2m), \tilde{x}}\}$$

where

$$\Theta^{(2m), \tilde{x}} = \left\{ X \in L^{2m}(\Omega, \mathcal{F}_T, P) : \exists \vartheta \in \mathcal{A}^{(2m)} \text{ s.t. } X = \tilde{x} + \int_0^T \vartheta_u dS_u \right\}.$$

Proof

► Verification

We have

$$E(u_{2m}(X^{(1)})) \geq E(u_{2m}(X^{(2)})) \geq E(u_{2m}(X^{(3)})) \geq E(u_{2m}(X^{(2m)}(\hat{Z}))).$$

A straightforward calculation shows that the convex dual of u_{2m} is given by

$$\check{u}_{2m}(y) = (2m - 1)y^{2m/(2m-1)} - 2my.$$

Standard duality theory can be applied to verify that $X^{(2m)}(\hat{Z})$ is the maximizer of problem Max_1 . All inequalities turn into identities. Moreover,

$$\begin{aligned} E(u_{2m}(X(\hat{Z}))) &= E(u_{2m}(X^{(2)})) \leq \inf_{Z \in \mathcal{D}_s^q, y \geq 0} (E(\check{u}_{2m}(y \cdot Z_T)) + \tilde{x}y) \\ &= \inf_{y \geq 0} \left((2m - 1)y^{2m/(2m-1)} \left(\inf_{Z \in \mathcal{D}_s^q} E[Z_T^{2m/(2m-1)}] \right) - (2m - \tilde{x})y \right) \end{aligned}$$

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