Modeling of the bank’s profitability via a Levy process-driven model and the Black Scholes model

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Outline

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Preliminaries

- Our *probability space* $(\Omega, \ \mathbf{F}, \ \mathbb{F}, \ \mathbf{P})$, are driven by a *Lévy process*.
- The *filtration* $\mathbb{F} = (\mathcal{F}_t)_{0 \leq t \leq \tau}$ is assumed to be the natural filtration of L.
- A Levy process $L = (L_t)_{0 \leq t \leq \tau}$ has *independent* and *stationary increments*.
- The *jump process* $\Delta L = (\Delta L_t, t \geq 0)$ associated to a Lévy process is defined by $\Delta L_t = L_t - L_{t^-}$.
- The *Lévy measure* ν satisfies

$$\int_{|x|<1} |x|^2 \nu(dx) < \infty, \quad \int_{|x|\geq 1} \nu(dx) < \infty.$$

Preliminaries

- σ_a is the volatility of the total assets, A.
- $\mu_a = \mu^g - \epsilon$ is the nett expected returns on A.
- σ_e is the volatility of the total equity, E.
- μ_e is the total expected returns on E

Bank's profitability measures

- Let $A^r = (A_t^r, t \geq 0)$ be the Lévy process of the **return on assets** (ROA) then

$$\text{ROA}\ (A^r) = \frac{\text{Net Profit After Taxes}}{\text{Assets.}}$$

- Let $E^r = (E_t^r, t \geq 0)$ be the Lévy process of the **return on equity** (ROE) then

$$ROE(E^r) = \frac{\text{Net Profit After Taxes}}{\text{Equity Capital.}}$$

Problem Statements

- To find the model that explain the dynamics of the **return on assets** (ROA) the best.
- To find the model that explain the dynamics of the **return on equity** (ROE) the best.

The Stochastic Banking Model

Our bank balance sheet:

$$\text{Value of } \textbf{Assets } (A) = \text{Value of } \textbf{Liabilities } (\Gamma) + \text{Value of } \textbf{Bank Capital } (C).$$

For the balance sheet identity (1), we can choose

$$A_t = \Lambda_t + R_t + S_t + B_t; \;\; \Gamma_t = D_t$$

where $\Lambda, \; R, \; S, \; B$ and D are the value of the corporate loans, reserves, marketable securities and treasuries and face value of the deposits, respectively. The value of the bank capital, $C = (C_t, t \geq 0)$ is constituded as follows

$$C_t = E_t + O_t$$

Black-Scholes model

We can express the dynamics of the value process of the:

- **TAs** A, **by means of the SDE**

$$dA_t = A_{t-}\Big[\mu_a dt + \sigma_a dZ_t^A\Big],$$

- **for the bank capital** $C = (C_t, t \geq 0)$:

$$dO_t = r\exp\{rt\}dt, \quad O_0 > 0$$

and:

$$dE_t \;=\; E_{t-}\Big[\mu_e dt + \sigma_e dZ_t^E\Big]$$

Merton's model

In Merton's model we get the *decomposition* of the Lévy process $L = (L_t)_{0 \leq t \leq \tau}$ into

$$L_t = at + \widetilde{s} Z_t + \sum_{i=1}^{N_t} Y_i, \quad 0 \leq t \leq \tau,$$

where

- $(Z_t)_{0 \leq t \leq \tau}$ is a BM with standard deviation $\widetilde{s} > 0$,
- $(N_t)_{t \geq 0}$ is a Poisson process counting the jumps
- $Y_i \sim N(\mu, \delta^2)$ are jumps sizes and $a = \mathbf{E}(L_1)$
- Put $\sigma^A = \widetilde{s}\sigma_a$ and $\mu^A = (\mu_a + a\sigma_a)$
- Put $\sigma^E = \widetilde{s}\sigma_e$ and $\mu^E = (\mu_e + a\sigma_e)$.

Merton's model

The dynamics of the value process of the

- TAs A,

$$dA_t = A_{t-}\Big[\mu^A dt + \sigma^A dZ_t^A + \sigma_a d[\sum_{i=1}^{N_t} Y_i]\Big],$$

- Bank capital:

$$dE_t = E_{t-}\Big[\mu^E dt + \sigma^E dZ_t^E + \sigma_e d[\sum_{i=1}^{N_t} Y_i]\Big]$$

- Net profit after tax:

$$d\Pi_t^n = \delta_e E_{t-}\Big[\mu^E dt + \sigma^E dZ_t^E + \sigma_e d[\sum_{i=1}^{N_t} Y_i]\Big] + \delta_s r \exp\{rt\}dt.$$

Dynamics ROA: Merton's case

$$
\begin{aligned}
dA_t^r = A_t^r \Big[& \Big(\delta_e E_t (\sigma^E)^2 \{(\sigma^A)^2 \sigma_a^2 dZ_t^A - \sigma_a^2\} + \sigma_a^2 \\
& + (\sigma^A)^2 - \mu^A + [\Pi_t^n]^{-1} \{\delta_e \mu^E E_t + \delta_e r O_t\}\Big) dt \\
& + \Big(d[\sum_{i=1}^{N_t} Y_i] \delta_e E_t \sigma^E \{\sigma^A \sigma_a dZ_t^A - \sigma_a\} \\
& + [\Pi_t^n]^{-1} \delta_e \sigma^E E_t \Big) dZ_t^E + \Big([\Pi_t^n]^{-1} \sigma^E \delta_e E_t + \sigma^A \sigma_a dZ_t^A - \sigma_a \\
& + \delta_e E_t \sigma^E [\Pi_t^n]^{-1} dZ_t^E \{\sigma^A \sigma_a dZ_t^A - \sigma_a\} \\
& - \delta_e E_t [\Pi_t^n]^{-1} \sigma^E \sigma^A dZ_t^A \Big) d[\sum_{i=1}^{N_t} Y_i] \\
& - \sigma_a \, dZ_t^A - \delta_e \sigma^A \sigma^E E_t [\Pi_t^n]^{-1} \, dZ_t^E \, dZ_t^A \Big].
\end{aligned}
$$

Dynamics ROE: Merton's case

$$
\begin{aligned}
dE_t^r = E_t^r\Big[& \Big([\Pi_t^n]^{-1}\Big\{\delta_e E_t \mu^E + \delta_s r O_t \\
& + \delta_e E_t(\sigma^E)^2\{2(\sigma^E)^2 + \sigma_e^2 dZ_t^E - \sigma_e^2\} \\
& -\delta_e E_t(\sigma^E)^2\Big\} + [\sigma^E]^2 - \mu_e + \sigma_e^2\Big)dt \\
& + \Big([\Pi_t^n]^{-1}\delta_e\sigma^E E_t - \sigma_e\Big)dZ_t^E \\
& + \Big([\Pi_t^n]^{-1}\sigma^E\delta_e E_t - \sigma_e + 2\sigma^E\sigma_e dZ_t^E\Big)d[\sum_{i=1}^{N_t} Y_i] \\
& + \Big(\delta_e E_t\sigma^E\{2\sigma^E\sigma_e dZ_t^E - \sigma_e\} \\
& - \delta_e E_t(\sigma^E)^2\Big)dZ_t^E d[\sum_{i=1}^{N_t} Y_i]\Big].
\end{aligned}
$$

Dynamics ROA: BS case

Special case where $L_t = Z_t$ i.e. $\sum_{i=1}^{N_t} Y_i + at = 0$.

$$
\begin{aligned}
dA_t^r \;&=\; A_t^r\Big[\{\sigma_a^2 - \mu_a + [\Pi_t^n]^{-1}(\delta_e \mu_e E_t + \delta_s r O_t)\}dt \\
&\quad + [\Pi_t^n]^{-1}\delta_e \sigma_e E_t \, dZ_t^E - \sigma_a \, dZ_t^A \\
&\quad - \delta_e \sigma_a \sigma_e E_t [\Pi_t^n]^{-1} \, dZ_t^E \, dZ_t^A\Big].
\end{aligned}
$$

Dynamics ROE: BS case

$$\begin{aligned} dE_t^r &= E_t^r \Bigg[\bigg([\sigma_e]^2 - \mu_e + [\Pi_t^n]^{-1} \Big\{ \delta_s r O_t + \delta_e E_t \mu_e \\ &\quad - \delta_e E_t (\sigma_e)^2 \Big\} \bigg) dt \\ &\quad + \bigg([\Pi_t^n]^{-1} \sigma_e \delta_e E_t - \sigma_e \bigg) dZ_t^E \Bigg]. \end{aligned}$$

Heston model

The **stochastic processes** for the ROA/ROE process and the variance process.

$$
\begin{aligned}
\frac{dS_t}{S_t} &= \mu\, dt + \sqrt{v_t}\, dW_{1t} \\
dv_t &= \kappa(\Theta - v_t)dt + \xi\sqrt{v_t}\, dW_{2t} \\
dW_{2t} &= \rho W_{1t} + \xi\sqrt{v_t} dW_{2t}
\end{aligned}
$$

Numerical Examples: ROA

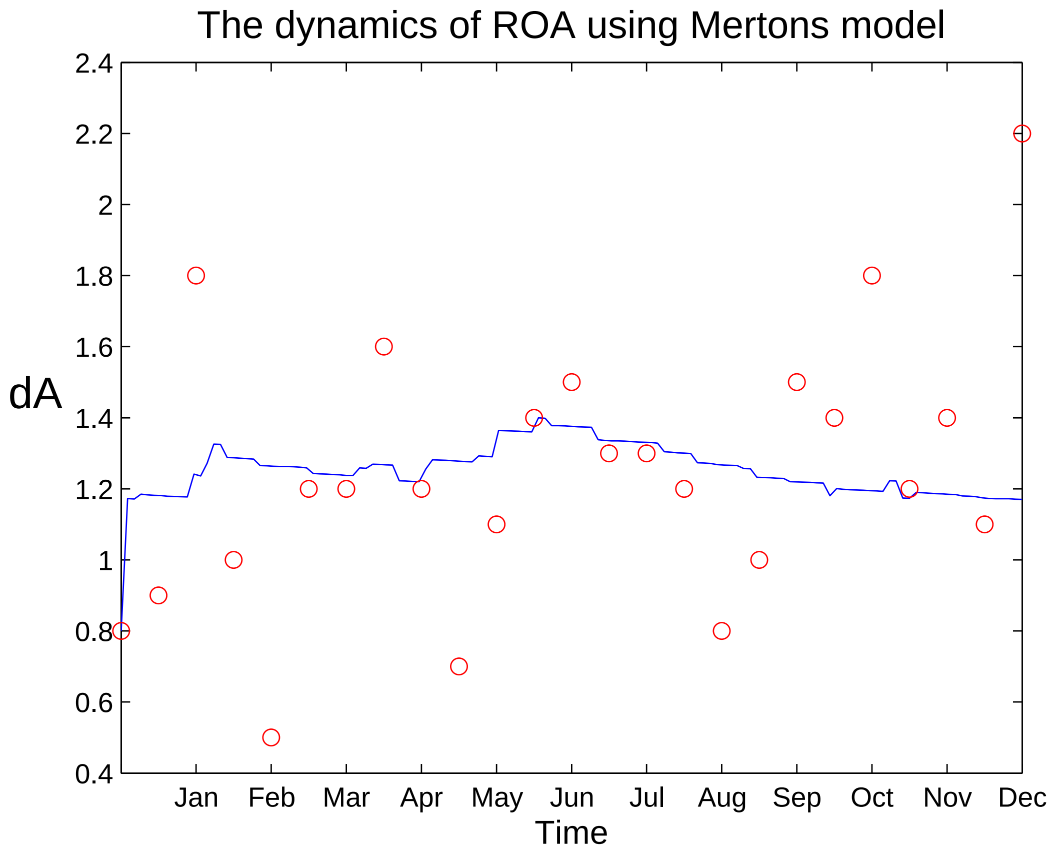
Jan-'05	0.9	Jan-'06	1.3	Jul-'05	1.6	Jul-'06	1.4
Feb-'05	1.8	Feb-'06	1.3	Aug-'05	1.2	Aug-'06	1.8
Mrt-'05	1	Mrt-'06	1.2	Sep-'05	.7	Sep-'06	1.2
Apr-'05	0.5	Apr-'06	0.8	Oct-'05	1.1	Oct-'06	1.4
May-'05	1.2	May-'06	1	Nov-'05	1.4	Nov-'06	1.1
Jun-'05	1.2	Jun-'06	1.5	Dec-'05	1.5	Dec-'06	2.2

The parameter choices

Parameter	**Simbol**	**Value**
Volatility of E_t	σ_e	2.55
Total expected returns on E_t	μ_e	0.12
Value of net profit after tax	Π_t^n	16878
Dividend payments on E	δ_e	0.05
Interest and principal payments on O	δ_s	1.06
Interest rate	r	0.06
Subordinate debt	O_t	135
Bank equity	E_t	1164
Volatility of A_t	σ_a	0.22
Nett expected returns on A_t	μ_a	0.01

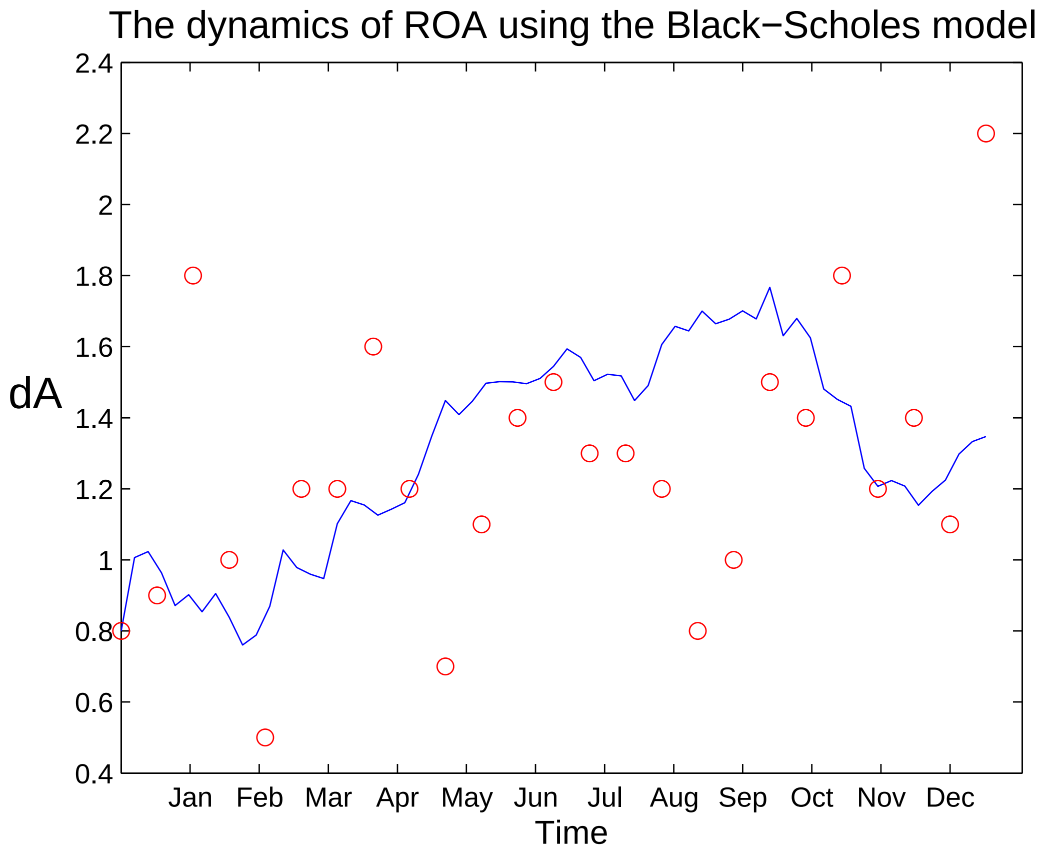
Figure 2. Parameter choices for the ROA simulation.

Merton

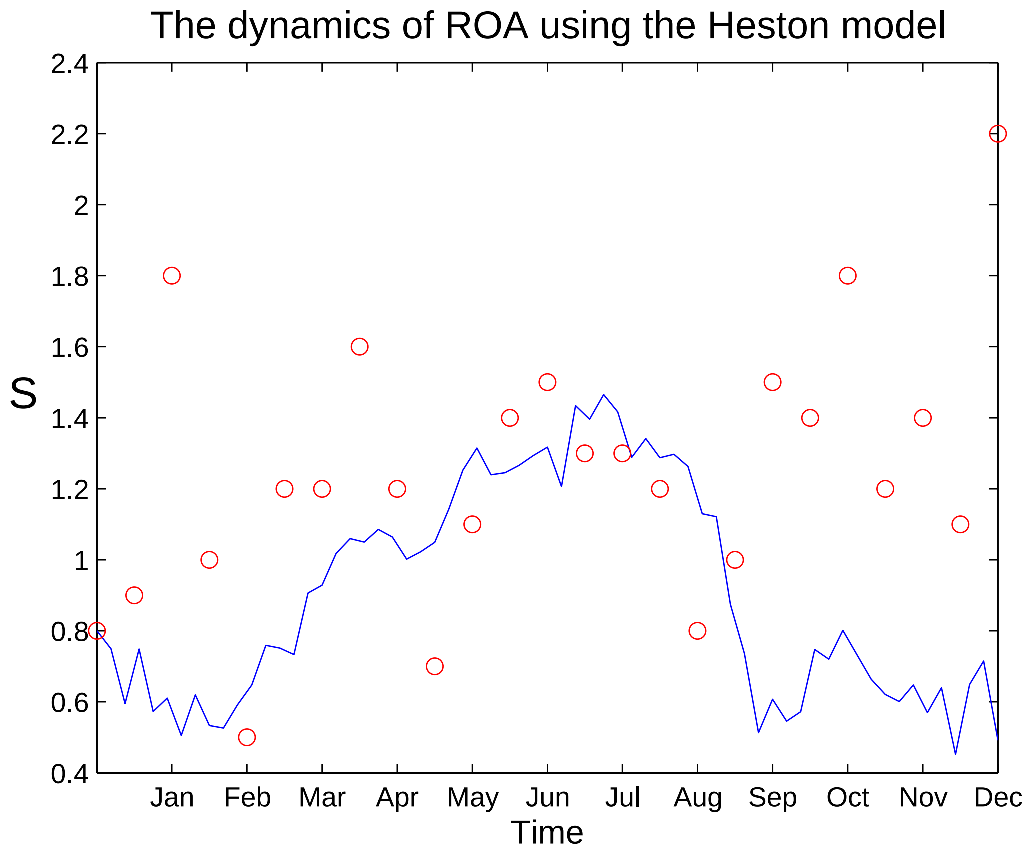


The dynamics of ROA using Mertons model
dA
Time
2.4
2.2
2
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
Jan
Feb
Mar
Apr
May
Jun
Jul
Aug
Sep
Oct
Nov
Dec

Black-Scholes



Heston


The dynamics of ROA using the Heston model
2.4
2.2
2
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
S
Jan
Feb
Mar
Apr
May
Jun
Jul
Aug
Sep
Oct
Nov
Dec
Time

Ongoing Research

- Descriptions of the dynamics of the other measures of bank profitability.
- A comprehensive financial interpretation of the results.