Lévy driven equity, FX- and interest rate models

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Abstract

Empirical analysis of data from the financial markets reveals that standard diffusion models
dS_t = \mu(t, S_t)dt + \sigma(t, S_t)dW_t
do not generate return distributions for stock prices, indices, or exchange rates with a sufficient degree of accuracy. To reduce model risk we introduce models of the form

S_t = S_0 \exp(X_t)

which are driven by Lévy processes or more general by semimartingales \((X_t)_{t \geq 0}\) instead of a Brownian motion \((W_t)_{t \geq 0}\). Analytical properties of this model class are investigated. For implementation in particular the class of generalized hyperbolic Lévy processes is considered. Plain vanilla as well as exotic options are priced on the basis of this model. As a further application in risk management we show that estimates of the value at risk of a portfolio of securities are improved.

In the second part we discuss Lévy term structure models. Three basic approaches to model interest rates are introduced: the forward rate model, the forward process model, and the LIBOR or market model. As an application pricing formulae for caps and floors are derived. Efficient algorithms to evaluate these formulae numerically are given. The LIBOR model can be extended to a multi-currency setting. Closed form pricing formulae for cross-currency derivatives such as foreign caps and floors and cross-currency swaps are studied in detail. The LIBOR model can also be extended to include defaultable instruments.
References


