Barrier Option Pricing Using Adjusted Transition Probabilities

G. Barone-Adesi   N. Fusari   J. Theal

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May 11th, 2007
1 Motivation
   - The standard Cox-Ross-Rubinstein Binomial Tree
   - Previous work

2 New Approach
   - Adjusted Binomial Tree

3 Results
   - Comparison
   - What is new?
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   - What is new?
Convergence behaviour of the Standard Tree

- The convergence of the standard Cox-Ross-Rubinstein Binomial Tree displays an erratic behaviour and a persistent bias in pricing Barrier Options.
- The reason for this behaviour, as pointed out by Boyle (1994), is that “the barrier will in general lie in between two adjacent nodes in the lattice.”
- If the barrier is near to the “starting node”, the standard method generates a relevant bias.
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Down-out European Call
Parameters: S=100, K=100, Vpl=25%, r=10%, T=1, Barrier=90

Option Price
Option Price (no adjustment)
Analytic Price

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Boyle and Lau (1994) (1)

Revised Binomial Tree

- In case of a constant barrier (H), it is easy to constrain the time partition such that the barrier lies just above a layer of horizontal nodes.
- Recall that, in the CRR model, the down (up) movement is equal to $d = \exp \left( -\sigma \sqrt{T/n} \right)$ ($u = \exp \left( \sigma \sqrt{T/n} \right)$).
- In order to obtain the desired result, it is enough to select $n$ (the number of steps in the tree) such that it is the largest integer smaller than

$$F(m) = \frac{m^2 \sigma^2 T}{\left( \ln S/H \right)^2} \quad m = 1, 2, \ldots \tag{1}$$

where $m$ is the number of down (up) steps that takes the asset price just above (below) the barrier.
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**Good news:**
- This method gives good approximation of the price of single and constant barrier options
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Ritchken (1995) (1)

“Stretched” Trinomial Tree

- Ritchken proposes an extension of the standard trinomial model
- As usual, the issue is to place the nodes as near as possible (from above) to the barrier
- Recall that, in the trinomial model, $up$ and $down$ movements are respectively $up = \exp(\lambda \sigma \sqrt{T/n})$ and $down = \exp(-\lambda \sigma \sqrt{T/n})$
- It is possible to select the stretch parameter $\lambda$ such that the barrier lies exactly on the nodes
- For more complex options, such as, options with double constant barrier, Ritchken proposes a state dependent tree with two stretch factors that allow to reposition the tree on the barriers
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Good news:
- This procedure gives a good approximation of the price of options with single (constant and time varying) and double (only constant) barriers.
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Bad news:
- Ritchken’s method does not provide approximation when the initial price of the underlying is close to the barrier.
- If a parameter of the option changes (maturity, volatility, etc...) then the entire lattice must be repositioned before calculating the new option price.
- It is not possible to price options with multiple time-varying barriers.
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Further Extensions

- Costabile (2001) proposes a discrete time algorithm for pricing double constant barrier options.

The problem with these procedures is that they are “ad hoc” solutions to specific cases; we look for a general algorithm that allows to approximate simple (single constant barrier) as well as complex (single and double time-varying) barriers options.
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Further Extensions

- Costabile (2001) proposes a discrete time algorithm for pricing double constant barrier options
- Costabile (2002) proposes an extension of the Cox-Ross-Rubinstein algorithm for pricing options with an exponential boundary

The problem with these procedures is that they are “ad hoc” solutions to specific cases; we look for a general algorithm that allows to approximate simple (single constant barrier) as well as complex (single and double time-varying) barriers options.
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   - Previous work

2 New Approach
   - Adjusted Binomial Tree

3 Results
   - Comparison
   - What is new?
Probability Adjustment

- Baldi, Caramellino and Iovino (1999) derive a series of approximations for the exit probability of the Brownian bridge that can be used to price multiple and time-varying barriers options.
- Recall that the log of the asset price follows a Brownian Motion.
- Baldi et al. use these probabilities to improve the Monte Carlo calculations, because with the standard procedure it is possible that the underlying asset price breaches the barrier without being detected.
- Here we use these probabilities to improve the Cox-Ross-Rubinstein binomial tree.
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Here we use these probabilities to improve the Cox-Ross-Rubinstein binomial tree.
Procedure

A Simple Example: Down-out call (DOC) with constant barrier (1)

The "adjusted tree" is similar to the standard CRR model; the difference relies on the computation of the value of the option in the nodes just above the barrier.
A Simple Example: Down-out call (DOC) with constant barrier (2)

That is, we multiply the usual probability of reaching the up point by one minus the conditional probability \( (p_L^\delta t) \) that the assets price, in the time interval, hits the barrier.
Motivation

New Approach

Results

Adjusted Binomial Tree

Procedure

A Simple Example: Down-out call (DOC) with constant barrier (3)

- \( p^\delta t_L \) = Exit Probability. This is the probability that the asset price, starting from \( S_{T_0} \) and ending in \( S_{T_0+\delta t}^{up} \), hits the barrier

- In this case we have

\[
p^\delta t_L (T_0, S_{T_0}, S_{T_0+\delta t}^{up}, L) = \exp \left( - \frac{2}{\sigma^2 \delta t} \right) \ln \left( \frac{S_{T_0}}{L} \right) \ln \left( \frac{S_{T_0+\delta t}^{up}}{L} \right)
\]

- With this probability adjustment the price of the DOC near the barrier (when \( S_{T_0+\delta t}^{down} < L \), at time \( T_0 \), is

\[
C_{DOC} = \exp(-r \delta t) p_{up}(1 - p^\delta t_L) C(S_{T_0+\delta t}^{up})
\]
Procedure

A Simple Example: Down-out call (DOC) with constant barrier (3)

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The same procedure can be extended to the pricing of more complex options with single and multiple time-varying barriers.

Baldi at al. provide exit probability approximations for all kinds of time-varying barriers.

In the paper we focus on:
- Single/double constant barriers
- Single/double exponential barriers
- Single/double (time) linear barriers

This simple procedure avoids the need of repositioning the tree “on the barrier”
Procedure

General Case

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Motivation

- The standard Cox-Ross-Rubinstein Binomial Tree
- Previous work

New Approach

- Adjusted Binomial Tree

Results

- Comparison
- What is new?
### Down-out European Call Price Approximation (1)

<table>
<thead>
<tr>
<th>Stk. P.</th>
<th>500</th>
<th>1000</th>
<th>2000</th>
<th>3000</th>
<th>4000</th>
<th>5000</th>
<th>Alyt. P.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(3.70)</td>
<td>(3.701)</td>
<td>(3.702)</td>
<td>(3.701)</td>
<td>(3.701)</td>
<td>(3.702)</td>
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</tr>
<tr>
<td>92.0</td>
<td>2.500</td>
<td>2.589</td>
<td>2.515</td>
<td>2.546</td>
<td>2.563</td>
<td>2.521</td>
<td>2.506</td>
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<tr>
<td></td>
<td>(2.504)</td>
<td>(2.506)</td>
<td>(2.506)</td>
<td>(2.506)</td>
<td>(2.506)</td>
<td>(2.506)</td>
<td></td>
</tr>
<tr>
<td>91.5</td>
<td>2.047</td>
<td>1.901</td>
<td>1.894</td>
<td>1.963</td>
<td>1.907</td>
<td>1.945</td>
<td>1.895</td>
</tr>
<tr>
<td></td>
<td>(1.894)</td>
<td>(1.894)</td>
<td>(1.895)</td>
<td>(1.895)</td>
<td>(1.895)</td>
<td>(1.895)</td>
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</tr>
<tr>
<td>91.0</td>
<td>1.242</td>
<td>1.365</td>
<td>1.263</td>
<td>1.331</td>
<td>1.315</td>
<td>1.279</td>
<td>1.274</td>
</tr>
<tr>
<td></td>
<td>—</td>
<td>(1.274)</td>
<td>(1.274)</td>
<td>(1.275)</td>
<td>(1.275)</td>
<td>(1.274)</td>
<td></td>
</tr>
<tr>
<td>90.5</td>
<td>0.810</td>
<td>0.758</td>
<td>0.624</td>
<td>0.663</td>
<td>0.691</td>
<td>0.699</td>
<td>0.642</td>
</tr>
<tr>
<td></td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>(0.642)</td>
</tr>
</tbody>
</table>

Table: $K=100$, Vol=25%, $r=10\%$, $T=1$, $L=90$, ($\circlearrowright$)=Ritchken’96.
# Down-out European Call Price Approximation (2)

<table>
<thead>
<tr>
<th>Stk. P.</th>
<th>500</th>
<th>1000</th>
<th>2000</th>
<th>3000</th>
<th>4000</th>
<th>5000</th>
<th>Alyt. P.</th>
</tr>
</thead>
<tbody>
<tr>
<td>90.4</td>
<td>0.649</td>
<td>0.642</td>
<td>0.576</td>
<td>0.508</td>
<td>0.521</td>
<td>0.537</td>
<td>0.515</td>
</tr>
<tr>
<td>90.3</td>
<td>0.476</td>
<td>0.490</td>
<td>0.479</td>
<td>0.450</td>
<td>0.419</td>
<td>0.390</td>
<td>0.387</td>
</tr>
<tr>
<td>90.2</td>
<td>0.303</td>
<td>0.316</td>
<td>0.327</td>
<td>0.328</td>
<td>0.323</td>
<td>0.316</td>
<td>0.258</td>
</tr>
<tr>
<td>90.1</td>
<td>0.142</td>
<td>0.146</td>
<td>0.152</td>
<td>0.156</td>
<td>0.159</td>
<td>0.161</td>
<td>0.129</td>
</tr>
<tr>
<td>90.05</td>
<td>0.068</td>
<td>0.069</td>
<td>0.071</td>
<td>0.072</td>
<td>0.073</td>
<td>0.074</td>
<td>0.065</td>
</tr>
<tr>
<td>90.01</td>
<td>0.013</td>
<td>0.013</td>
<td>0.013</td>
<td>0.013</td>
<td>0.013</td>
<td>0.013</td>
<td>0.013</td>
</tr>
</tbody>
</table>

Table: $K=100$, Vol=25%, $r=10\%$, $T=1$, $L=90$, ( )=Ritchken’96.
Convergence Behaviour

Double Knock-out European Call with Constant Barriers

![Graph showing convergence behaviour](image-url)

**Double knock-out European Call Option**
Parameters: S=100, K=80, Vol=25%, r=10%, T=1, U=120, L=85

[Graph showing convergence behaviour with parameters listed above]
Convergence Behaviour

Double Knock-out European Call with Constant Barriers

Double knock-out European Call Option
Parameters: S=100, K=80, Vol=25%, r=10%, T=1, U=120, L=85

Option Price (no adjustment)
Option Price (with adjustment)
Barrier distance measure

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Convergence Behaviour
Down-out European Call with Exponential Barrier

Parameters: S=95, K=100, Vol=25%, r=10%, T=1, Slope=0.05, Intercept=90

Option Price
Option Price (no adjustment)
Analytic Price
Option Price (with adjustment)
Convergence Behaviour
Down-out European Call with Exponential Barrier

Parameters: S=95, K=100, Vol=25%, r=10%, T=1, Slope=0.05, Intercept=90

Option Price
Option Price (no adjustment)
Analytic Price
Option Price (with adjustment)
Convergence Behaviour
Down-out European Call with Linear Barrier

Parameters: $S=100$, $K=100$, $Vol=25\%$, $r=10\%$, $L_0=95$, $L_1=10$

![Graph showing convergence behaviour of down-out European call with linear barrier.](image)

Option Price (no adjustment) vs. Option Price (with adjustment) vs. Barrier distance measure.
Convergence Behaviour
Down-out European Call with Linear Barrier

Down-out European Call with Linear Barrier
Parameters: S=100, K=100, Vol=25%, r=10%, L0=95, L1=10
Convergence Behaviour
Double Knock-out European Call with Linear Barriers

Parameters: S=100, K=100, Vol=25%, r=10%, L0=92, L1=-22, U0=105, U1=35

Option Price
Option Price (no adjustment)
Option Price (with adjustment)
Convergence Behaviour
Double Knock-out European Call with Linear Barriers

Parameters: S=100, K=100, Vol=25%, r=10%, L0=92, L1=-22, U0=105, U1=35

Option Price
Option Price (no adjustment)
Option Price (with adjustment)
### Double Knock-out European Call with Short Maturity

<table>
<thead>
<tr>
<th>Vol</th>
<th>U</th>
<th>L</th>
<th>KI</th>
<th>FD</th>
<th>Approx1</th>
<th>Approx2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma = 0.20$</td>
<td>1500</td>
<td>500</td>
<td>25.12</td>
<td>24.47</td>
<td>25.12</td>
<td>25.12</td>
</tr>
<tr>
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Table: $U =$ Upper Barrier, $L =$ Lower Barrier, $KI =$ Kunitomo and Ikeda method, $FD =$ Finite Difference method, Approx1 = 1000 time-divisions, Approx2 = 2000 time-divisions. Parameters: $S_0 = 1000, K = 1000, r = 5\%, T = 0.5$. 

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G. Barone-Adesi, N. Fusari, J. Theal

Barrier Option Pricing Using Adjusted Transition Probabilities
### Down-out European Call with Exponential Barrier

<table>
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**Table:** Comparison of results between the adjusted-probability method and the extended Cox-Ross-Rubinstein method of Costabile (2002)
Double Knock-out European Call

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<th>Tree Lvl.</th>
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<td>0.0765</td>
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<tr>
<td>5000</td>
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<td>Analyt. Value</td>
<td>0.0411</td>
<td>0.0178</td>
<td>0.0762</td>
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</table>

Table: Comparison of results between the adjusted-probability method and the analytical values calculated by Geman and Yor (1996).

Case 1: \( S_0 = 2, K = 2, \sigma = 20\%, r = 2\%, T = 1\text{ year}, L = 1.5, U = 2.5 \)

Case 2: \( S_0 = 2, K = 2, \sigma = 50\%, r = 5\%, T = 1\text{ year}, L = 1.5, U = 3 \)

Case 3: \( S_0 = 2, K = 1.75, \sigma = 50\%, r = 5\%, T = 1\text{ year}, L = 1.5, U = 3 \)
Motivation

- The standard Cox-Ross-Rubinstein Binomial Tree
- Previous work

New Approach

- Adjusted Binomial Tree

Results

- Comparison
- What is new?
Goals of the Paper

The “Adjusted Tree”:

- does not require to reposition the tree “on the barrier”
- demonstrates good convergence properties towards the analytical price for both single and double constant barriers options
- produces an accurate approximation when the stock price is very close to the barrier
- produces an accurate approximation for options with short-term maturity
- can produce price approximation for time-varying barrier options including exponential, single linear and double linear barriers
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