Modeling of the bank’s profitability via a Levy process-driven model and the Black Scholes model

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Outline

• Preliminaries
• The two main measures of a bank’s profitability
• Problem statements
• The stochastic banking model
• The Black-Scholes model
• Merton’s model: Levy process-driven model
• The dynamics of the ROA and the ROE
• Numerical examples
• Ongoing research
Preliminaries

- Our probability space \((\Omega, \mathcal{F}, \mathbb{F}, \mathbb{P})\), are driven by a Lévy process.
- The filtration \(\mathbb{F} = (\mathcal{F}_t)_{0 \leq t \leq \tau}\) is assumed to be the natural filtration of \(L\).
- A Lévy process \(L = (L_t)_{0 \leq t \leq \tau}\) has independent and stationary increments.
- The jump process \(\Delta L = (\Delta L_t, t \geq 0)\) associated to a Lévy process is defined by \(\Delta L_t = L_t - L_{t-}\).
- The Lévy measure \(\nu\) satisfies

\[
\int_{|x| < 1} |x|^2 \nu(dx) < \infty, \quad \int_{|x| \geq 1} \nu(dx) < \infty.
\]
Preliminaries

• $\sigma_a$ is the volatility of the total assets, $A$.
• $\mu_a = \mu^g - \epsilon$ is the nett expected returns on $A$.
• $\sigma_e$ is the volatility of the total equity, $E$.
• $\mu_e$ is the total expected returns on $E$
Bank’s profitability measures

- Let $A^r = (A^r_t, t \geq 0)$ be the Lévy process of the return on assets (ROA) then

$$\text{ROA} (A^r) = \frac{\text{Net Profit After Taxes}}{\text{Assets}}.$$ 

- Let $E^r = (E^r_t, t \geq 0)$ be the Lévy process of the return on equity (ROE) then

$$\text{ROE} (E^r) = \frac{\text{Net Profit After Taxes}}{\text{Equity Capital}}.$$
Problem Statements

- To find the model that explain the dynamics of the **return on assets** (ROA) the best.
- To find the model that explain the dynamics of the **return on equity** (ROE) the best.
The Stochastic Banking Model

Our bank balance sheet:

Value of Assets \((A)\) = Value of Liabilities \((\Gamma)\) + Value of Bank Capital \((C)\).

For the balance sheet identity (1), we can choose

\[ A_t = \Lambda_t + R_t + S_t + B_t; \quad \Gamma_t = D_t \]

where \(\Lambda, R, S, B\) and \(D\) are the value of the corporate loans, reserves, marketable securities and treasuries and face value of the deposits, respectively. The value of the bank capital, \(C = (C_t, t \geq 0)\) is constituted as follows

\[ C_t = E_t + O_t \]
Black-Scholes model

We can express the dynamics of the value process of the:

- **TAs** $A$, by means of the SDE

  $$dA_t = A_t - \left[ \mu_a \, dt + \sigma_a \, dZ^A_t \right],$$

- **for the bank capital** $C = (C_t, t \geq 0)$:

  $$dO_t = r \exp\{rt\} \, dt, \quad O_0 > 0$$

  and:

  $$dE_t = E_t - \left[ \mu_e \, dt + \sigma_e \, dZ^E_t \right]$$
Merton’s model

In Merton’s model we get the decomposition of the Lévy process $L = (L_t)_{0 \leq t \leq \tau}$ into

$$L_t = at + \tilde{s}Z_t + \sum_{i=1}^{N_t} Y_i, \quad 0 \leq t \leq \tau,$$

where

- $(Z_t)_{0 \leq t \leq \tau}$ is a BM with standard deviation $\tilde{s} > 0$,
- $(N_t)_{t \geq 0}$ is a Poisson process counting the jumps
- $Y_i \sim N(\mu, \delta^2)$ are jumps sizes and $a = \mathbb{E}(L_1)$
- Put $\sigma^A = \tilde{s}\sigma_a$ and $\mu^A = (\mu_a + a\sigma_a)$
- Put $\sigma^E = \tilde{s}\sigma_e$ and $\mu^E = (\mu_e + a\sigma_e)$. 
Merton’s model

The dynamics of the value process of the

- TAs $A$,

$$dA_t = A_t\left[ \mu^A dt + \sigma^A dZ_t^A + \sigma_a d\left[ \sum_{i=1}^{N_t} Y_i \right] \right],$$

- Bank capital:

$$dE_t = E_t\left[ \mu^E dt + \sigma^E dZ_t^E + \sigma_e d\left[ \sum_{i=1}^{N_t} Y_i \right] \right],$$

- Net profit after tax:

$$d\Pi_t^n = \delta_e E_t\left[ \mu^E dt + \sigma^E dZ_t^E + \sigma_e d\left[ \sum_{i=1}^{N_t} Y_i \right] \right] + \delta_s r \exp\{rt\} dt.$$
Dynamics ROA: Merton’s case

\[ dA_t^r = A_t^r \left[ \left( \delta_e E_t (\sigma^E)^2 \{ (\sigma^A)^2 \sigma_a^2 dZ_t^A - \sigma_a^2 \} + \sigma_a^2 \right. \right. \]
\[ + (\sigma^A)^2 - \mu^A + [\Pi_t^n]^{-1} \{ \delta_e \mu^E E_t + \delta_e rO_t \} \right) dt \]
\[ + \left( \left. d\left[ \sum_{i=1}^{N_t} Y_i \right] \delta_e E_t \sigma^E \{ \sigma^A \sigma_a dZ_t^A - \sigma_a \} \right. \right. \]
\[ + [\Pi_t^n]^{-1} \delta_e \sigma^E E_t \right) dZ_t^E + \left( [\Pi_t^n]^{-1} \sigma^E \delta_e E_t + \sigma^A \sigma_a dZ_t^A - \sigma_a \right) \]
\[ + \delta_e E_t \sigma^E [\Pi_t^n]^{-1} dZ_t^E \{ \sigma^A \sigma_a dZ_t^A - \sigma_a \} \]
\[ - \delta_e E_t [\Pi_t^n]^{-1} \sigma^E \sigma^A dZ_t^A \right) \left. d\left[ \sum_{i=1}^{N_t} Y_i \right] \right. \]
\[ - \sigma_a dZ_t^A - \delta_e \sigma^A \sigma^E E_t [\Pi_t^n]^{-1} dZ_t^E dZ_t^A \].
Dynamics ROE: Merton’s case

\[ dE_t^r = E_t^r \left[ \left( \Pi_t^n \right)^{-1} \left\{ \delta_e E_t \mu^E + \delta_s r O_t \right\} + \delta_e E_t (\sigma^E)^2 \left\{ 2(\sigma^E)^2 + \sigma_e^2 dZ_t^E - \sigma_e^2 \right\} \\
- \delta_e E_t (\sigma^E)^2 \right\} + [\sigma^E]^2 - \mu_e + \sigma_e^2 \right] dt \\
+ \left( \left( \Pi_t^n \right)^{-1} \delta_e \sigma^E E_t - \sigma_e \right) dZ_t^E \\
+ \left( \left[ \Pi_t^n \right]^{-1} \sigma^E \delta_e E_t - \sigma_e + 2\sigma^E \sigma_e dZ_t^E \right) d[\sum_{i=1}^{N_t} Y_i] \\
+ \left( \delta_e E_t \sigma^E \left\{ 2\sigma^E \sigma_e dZ_t^E - \sigma_e \right\} \\
- \delta_e E_t (\sigma^E)^2 \right) dZ_t^E d[\sum_{i=1}^{N_t} Y_i]. \]
Dynamics ROA: BS case

Special case where $L_t = Z_t$ i.e.

$$\sum_{i=1}^{N_t} Y_i + at = 0.$$ 

$$dA_t = A_t \left[ \left\{ \sigma_a^2 - \mu_a + [\Pi_t^n]^{-1}(\delta e \mu_e E_t + \delta s r O_t) \right\} dt 
+ [\Pi_t^n]^{-1} \delta e \sigma_e E_t dZ^E_t - \sigma_a dZ^A_t 
- \delta e \sigma_a \sigma_e E_t [\Pi_t^n]^{-1} dZ^E_t dZ^A_t \right].$$
Dynamics ROE: BS case

\[ dE_t^r = E_t^r \left[ \left( [\sigma_e]^2 - \mu_e + [\Pi_t^n]^{-1} \left\{ \delta_s r O_t + \delta_e E_t \mu_e 
- \delta_e E_t (\sigma_e)^2 \right\} \right) dt 
+ \left( [\Pi_t^n]^{-1} \sigma_e \delta_e E_t - \sigma_e \right) dZ_t^E \right]. \]
Heston model

The stochastic processes for the ROA/ROE process and the variance process.

\[
\frac{dS_t}{S_t} = \mu dt + \sqrt{v_t} \, dW_{1t} \\
dv_t = \kappa (\Theta - v_t) dt + \xi \sqrt{v_t} \, dW_{2t} \\
dW_{2t} = \rho W_{1t} + \xi \sqrt{v_t} \, dW_{2t}
\]
### Numerical Examples: ROA

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<th>Year 1</th>
<th>Month 2</th>
<th>Year 1</th>
<th>Month 2</th>
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<td>Dec-’05</td>
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The parameter choices

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<th>Parameter</th>
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<td>Nett expected returns on $A_t$</td>
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</table>

Figure 2. Parameter choices for the ROA simulation.
The dynamics of ROA using Merton's model

- Time
- dA

- Jan
- Feb
- Mar
- Apr
- May
- Jun
- Jul
- Aug
- Sep
- Oct
- Nov
- Dec
The dynamics of ROA using the Black–Scholes model
The dynamics of ROA using the Heston model
**Ongoing Research**

- Descriptions of the dynamics of the other measures of bank profitability.
- A comprehensive financial interpretation of the results.