1a) $\;{}_tP_x = \exp\left(-\int_x^{x+t} \mu_s\,ds\right)$

$x \mapsto \int_x^{x+t} \mu_s\,ds$ monoton wachsend (Ableitung $\mu_{x+t} - \mu_x$)

$x \mapsto e^{-x}$ monoton fallend

$\Rightarrow_{x \mapsto} {}_tP_x$ mon. fallend $\forall t>0$

$\Rightarrow E(T_x) = \int_0^\infty {}_tP_x\,dt$ mon. wachsend in x

1b) $\;E(T_x) = \int_0^\infty {}_tP_x\,dt = \int_0^\infty \exp\left(-t\left(\frac{10+x}{1000}\right)\right)dt = -\frac{1000}{10+x}\left(\exp\left(-t\left(\frac{10+x}{1000}\right)\right)\right)\Big|_0^\infty$

$= \frac{10+x}{1000}$

$E(T_x^2) = \int_0^\infty 2t\,{}_tP_x\,dt = 2\int_0^\infty t\exp\left(-t\left(\frac{10+x}{1000}\right)\right)dt$

$= 2\left(-\frac{1000}{10+x}\,t\,\exp\left(-t\left(\frac{10+x}{1000}\right)\right) - \frac{1000^2}{(10+x)^2}\exp\left(-t\left(\frac{10+x}{1000}\right)\right)\right)\Big|_0^\infty$

$= 2\left(-0 - 0 + 0 + \frac{1000^2}{(10+x)^2}\right) = 2\,\frac{1000^2}{(10+x)^2}$

$\Rightarrow Var(T_x) = \left(\frac{1000}{10+x}\right)^2$

1c) $\;X$ = Nettoeinmalprämie = Barwert der Leistung der Versicherung

$X = E\left(v^{T_{40}} e^{\frac{40+T_{40}-40}{20}} \mid T_{40} \in [0,20]\right) P(T_{40} \in [0,20]) + X \cdot P(T_{40} > 20) \cdot v^{20}$

$= \frac{50}{1000}\int_0^{20} \underbrace{e^{\frac{\cancel{t}}{20}} \cdot e^{-\delta\cancel{t}} \cdot e^{-\frac{50}{1000}\cdot t}}_{e^{-\frac{1}{20}t}}\,dt + X\int_{20}^\infty e^{-\frac{50\cdot t}{1000}} \cdot \frac{50}{1000}\,dt \cdot \underbrace{e^{-20\delta}}_{e^{-1}}$

$= -\left[e^{-\frac{1}{20}t}\right]_0^{20} + X \cdot \left[e^{-\frac{t}{20}}\right]_{20}^\infty \cdot e^{-1}$

$= -\left(e^{-1} - 1\right) + X \cdot \left(0 - e^{-1}\right)e^{-1} = 1 + Xe^{-2} - e^{-1}$

$\Rightarrow X(1 - e^{-2}) = 1 - e^{-1} \Rightarrow X = \frac{1 - e^{-1}}{1 - e^{-2}}$

2a, $v = \frac{1}{1.03}$

T_{81}	0	1	2	3	≥ 4
P	$9/93$	$11/93$	$13/93$	$15/93$	$43/93$
Z	v	v^2	v^3	v^4	v^4

$\Rightarrow NEP = \frac{1}{93}\left(9v + 11v^2 + 13v^3 + 58v^4\right) = 0.8875$

2b $NEP = \sum_{k=1}^{4} kc \cdot v^{k-1} \cdot {}_{k-1}p_{81} = c \cdot 1 + 2cv \cdot \frac{82}{93} + 3cv^2 \cdot \frac{71}{93} + 4cv^3 \frac{58}{93}$

$\Rightarrow c = \frac{0.8875}{1 + 2v\frac{82}{93} + 3v^2\frac{71}{93} + 4v^3 \cdot \frac{58}{93}} = 0.124$

2c $x = 84$

Deckungskapital = ~~[illegible]~~ = v (Es wird garantiert 1€ in Einem Jahr fällig)

Nachschüssig Leibrente $a_{84} = v \cdot \frac{43}{58} + v^2 \cdot \frac{26}{58} + v^3 \cdot \frac{7}{58} = 1.2528$

$\Rightarrow$ Ausbezahlter Betrag = $\frac{\cancel{[illegible] \cdot 1.25}}{1.2528} = \cancel{0.7741}$

$= \frac{v}{1.2528} = 0.7750$

3a)

$$p_{65} = {}_1p_{65} = \frac{P(T_{60} \geq 6)}{P(T_{60} \geq 5)} = \frac{1-12\%}{1-10\%} = \frac{88}{90} \approx 0.9778$$

$$p_{73} = \frac{P(T_{70} \geq 4)}{P(T_{70} \geq 3)} = \frac{1-16\%}{1-12\%} \approx 0.9545$$

3b) weil der ~~Auszahl~~ Einzahlung:

$$\int_0^{\min(T_{60},10)} v^t \cdot p \cdot dt = \int_0^{10} 0.02 \cdot \int_0^t v^s \cdot p \, ds \, dt + 0.8 \cdot \int_0^{10} p v^s ds =$$

$$0.02p \int_0^{10} \frac{v^t - 1}{\ln v} dt + 0.8\, p \frac{v^{10}-1}{\ln v} = \frac{0.02p}{\ln v} \cdot \frac{v^{10}-1}{\ln v} - \frac{0.02p}{\ln v} \cdot 10 + \frac{0.8p}{\ln v}(v^{10}-1)$$

$$\approx p \cdot 7{,}1072$$

Auszahlung:

$$v^{10} {}_{10}p_{60} \int_0^{\min(10,T_{70})} v^t dt \overset{\text{s.o.}}{=} v^{10} 0.8 \left(\frac{0.04}{\ln v} \cdot \frac{v^{10}-1}{\ln v} - \frac{0.04p}{\ln v} \cdot 10 + \frac{0.6}{\ln v}(v^{10}-1) \right)$$

$$\approx 6.3915 \cdot v^{10} \cdot 0.8$$

$$\Rightarrow p = \frac{6.3915 \cdot v^{10} \cdot 0.8}{7.1072 \cdot \cancel{v^{10} \cdot 0.8}} \approx \underline{0.4308}$$

3c) statt 1 € muss man $\frac{1}{0{,}95}$ € einzahlen

$$\Rightarrow p_{neu} = \frac{p_{alt}}{0.95} = 0.4534$$

3d) wie in 3b (Prospektiv)

$$\int_0^{\min(10,T_{70})} v^t dt = 6.3915$$